Bjerknes Forces

Another force that can be important for bubbles is that experienced by a bubble placed in an acoustic field. Termed the Bjerknes force, this non-linear effect results from the the finite wavelength of the sound waves in the liquid. The frequency, wavenumber, and propagation speed of the stationary acoustic field will be denoted by ω, κ and c_L respectively where $\kappa = \omega/c_L$. The finite wavelength implies an instantaneous pressure gradient in the liquid and, therefore, a *buoyancy* force acting on the bubble.

To model this we express the instantaneous pressure, p by

$$p = p_o + Re\{\tilde{p}^* \sin(\kappa x_i) e^{i\omega t}\} \tag{Nfe1}$$

where p_o is the mean pressure level, $\tilde{p}^*$ is the amplitude of the sound waves and x_i is the direction of wave propagation. Like any other pressure gradient, this produces an instantaneous force, F_i, on the bubble in the x_i direction given by

$$F_i = -\frac{4}{3}\pi R^3 \left(\frac{dp}{dx_i}\right) \tag{Nfe2}$$

where R is the instantaneous radius of the spherical bubble. Since both R and dp/dx_i contain oscillating components, it follows that the combination of these in equation (Nfe2) will lead to a nonlinear, time-averaged component in F_i, that we will denote by $\bar{F}_i$. Expressing the oscillations in the volume or radius by

$$R = R_e \left[1 + Re\{\varphi e^{i\omega t}\}\right] \tag{Nfe3}$$

one can use the Rayleigh-Plesset equation (see section (Ngb)) to relate the pressure and radius oscillations and thus obtain

$$Re\{\varphi\} = \frac{\tilde{p}^*(\omega^2 - \omega_n^2)\sin(\kappa x_i)}{\rho_L R_e^2 \left[(\omega^2 - \omega_n^2)^2 + \left(4\nu_L \omega / R_e^2\right)^2\right]} \tag{Nfe4}$$

where ω_n is the natural frequency of volume oscillation of an individual bubble (see section (Ngj)) and μ_L is the effective viscosity of the liquid in damping the volume oscillations. If ω is not too close to ω_n, a useful approximation is

$$Re\{\varphi\} \approx \tilde{p}^* \sin(\kappa x_i)/\rho_L R_e^2(\omega^2 - \omega_n^2) \tag{Nfe5}$$

Finally, substituting equations (Nfe1), (Nfe3), (Nfe4), and (Nfe5) into (Nfe2) one obtains

$$\bar{F}_i = -2\pi R_e^3 Re\{\varphi\}\kappa\tilde{p}^* \cos(\kappa x_i) \approx -\frac{\pi\kappa R_e (\tilde{p}^*)^2 \sin(2\kappa x_i)}{\rho_L(\omega^2 - \omega_n^2)} \tag{Nfe6}$$

This is known as the primary Bjerknes force since it follows from some of the effects discussed by that author (Bjerknes 1909). The effect was first properly identified by Blake (1949).

The form of the primary Bjerknes force produces some interesting bubble migration patterns in a stationary sound field. Note from equation (Nfe6) that if the excitation frequency, ω, is less than the bubble natural frequency, ω_n, then the primary Bjerknes force will cause migration of the bubbles away from the nodes in the pressure field and toward the antinodes (points of largest pressure amplitude). On the other hand, if $\omega > \omega_n$ the bubbles will tend to migrate from the antinodes to the nodes. A number of investigators (for example, Crum and Eller 1970) have observed the process by which small bubbles in a stationary sound field first migrate to the antinodes, where they grow by rectified diffusion (see section (Ngl)) until they are larger than the resonant radius. They then migrate back to the nodes, where they may dissolve again when they experience only small pressure oscillations. Crum and Eller (1970) and have shown that the translational velocities of migrating bubbles are compatible with the Bjerknes force estimates given above.