Boundary Layers

In general, a boundary layer in a fluid or solid is identified as the layer next to the boundary in which the fluid properties have been affected by the presence of the boundary. In the thermal sciences, it usually refers to the layer in which the temperature, the fluid velocity or concentration differ from that of the bulk because of the presence of the boundary. In this section we address the fluid velocity boundary layers in flows with Reynolds numbers substantially greater than unity. The reason for confining the analysis to flows with Reynolds numbers substantially greater than unity will become clear shortly.

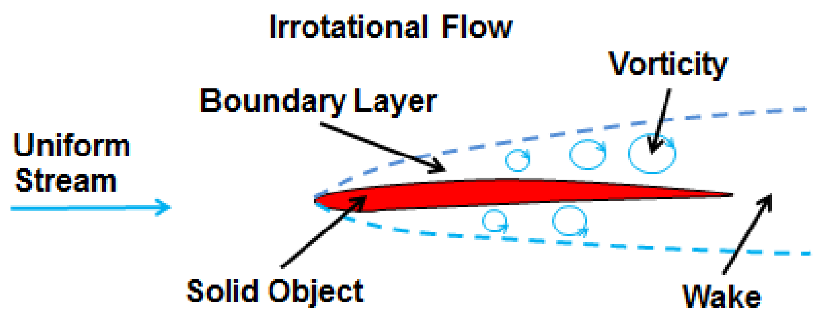

Figure 1: Typical boundary layer in a high Reynolds number flow

Consider, for example, the flow around an airfoil as depicted in Figure 1. As discussed in the section on vorticity, there is a thin boundary layer next to the surface of the airfoil (at least in high Reynolds number flows) within which the flow is **not** irrotational. Outside of this layer the flow is irrotational since the uniform flow upstream is irrotational and the vorticity that is created by the no-slip condition at the solid surface cannot diffuse out beyond the boundary layer because it is also being convected downstream. We can estimate the thickness of the boundary layer in steady flow as follows. Consider an element of fluid that passes the leading edge of the foil at time $t = 0$; at a later time t the vorticity will have diffused out from the surface a distance of $(\nu t)^{1/2}$ while at the same time the fluid will have been convected a distance $s = Ut$ along the surface. It follows that the boundary layer thickness, δ (defined as the maximum distance that the vorticity has diffused away from the surface), in steady flow will be given by $\delta = (\nu t)^{1/2}$ where $t = s/U$ and therefore

$$\delta = (\nu s/U)^{1/2} \qquad \text{or} \qquad \frac{\delta}{s} = \frac{1}{(Us/\nu)^{1/2}} = Re_s^{-1/2} \tag{Bja1}$$

where Re_s is the Reynolds number based on the freestream velocity, U, and the distance s along the surface of the foil. Consequently, it follows in retrospect that if the Reynolds number, Re_L, based on the chord length, L, is much greater than one (and therefore all relevant values of Re_s are much greater than unity except very close to the leading edge) then the boundary layer will be much thinner than the chord, L, and the the vorticity in the flow exterior to the boundary layer will be very small. Conversely, it also follows that if the Reynolds number were smaller than unity the vorticity would spread throughout the flow and no boundary would exist.

In the next section we will develop the equations governing the velocity components in a *laminar* incompressible boundary layer on a body in steady flow. The issues of the stability of that laminar flow and how and when it may become unstable and evolve into a turbulent boundary layer will be delayed until

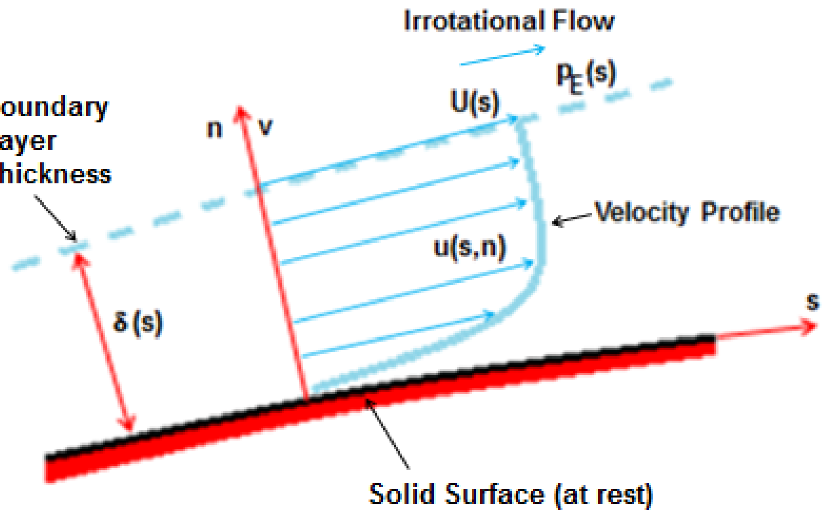

Figure 2: Depiction of the velocity profile in a boundary layer in a steady, planar flow

later. It is also convenient to begin by detailing the equations for a boundary layer in a *steady, planar, incompressible* flow. To proceed we first establish the notation that is utilized in this derivation. As depicted in Figure 2, it is convenient to define a set of coordinates, (s, n), in which s is measured along the solid boundary surface (it is assumed that a transformation has been applied to bring that surface to rest) and n is measured normal to and out from that surface (the boundary curvature is assumed small and its effects neglected in this initial analysis. The corresponding velocities in the s and n directions are denoted by $u(s, n)$ and $v(s, n)$ respectively. The velocity, $u(s, n)$, within the boundary layer defines the boundary layer *velocity profile* (see Figure 2) which is a key feature of the layer and whose shape and properties determine the evolution of the flow. Note, in addition, that the boundary layer thickness, $\delta(s)$, has yet to be precisely defined and that the velocity at the edge of the boundary layer, $U(s)$, is assumed to be given from knowledge of the flow outside the layer. The issue of the extent to which that external flow may be effected by the boundary layer is left until later; for the present it is assumed that the external flow is known and that the velocity, U, and pressure, p_E, on the edge of the boundary layer are known *a priori*.

Before leaving this introduction, it is important to stress that there are two substantial complications that can occur in a laminar boundary layer and are disregarded for the moment in order to develop a basic understanding of laminar boundary layers. These two complications are (1) instability within the laminar boundary layer leading to a turbulence and a turbulent boundary layer and (2) seperation of the laminar boundary layer. We will return later to descriptions and analyses of both of these important phenomena.