Oblique Shock Wave

Figure 1 depicts a large compression deflection and clearly demonstrates why this is different from a large expansion deflection. The Mach waves from a gradual compression deflection will intersect forming a

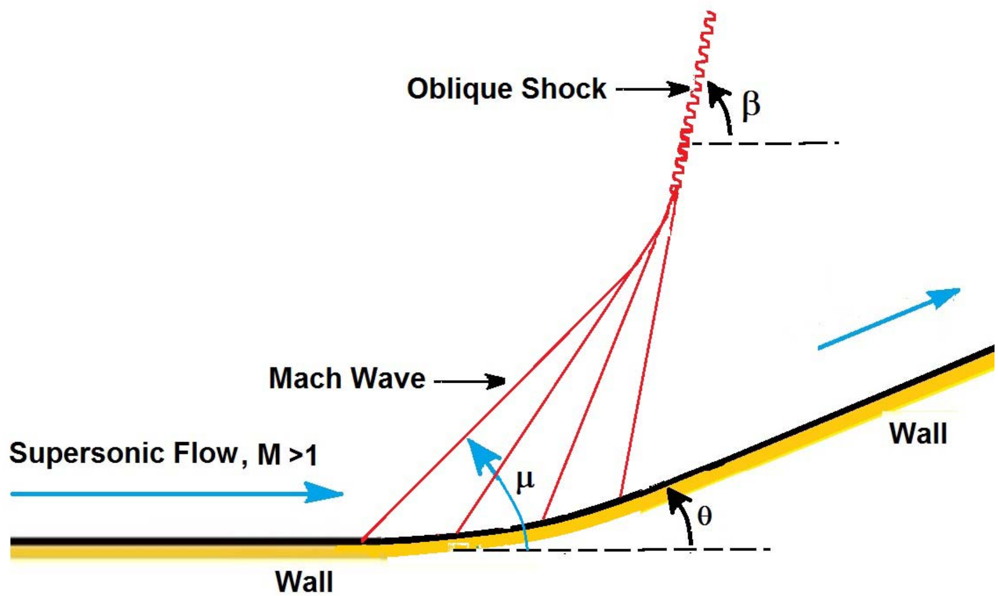

Figure 1: Supersonic flow performing a large compressive deflection, θ.

caustic across which the changes in the velocity, Mach number, pressure, temperature and density are no longer small. This caustic structure is an *oblique shock wave* and when the flow is viewed from a perspective far from the deflection, this oblique shock wave appears to emanate from the vertex as depicted in Figure 2. Moreover, since the changes across the oblique shock wave are no longer small, this feature of the flow

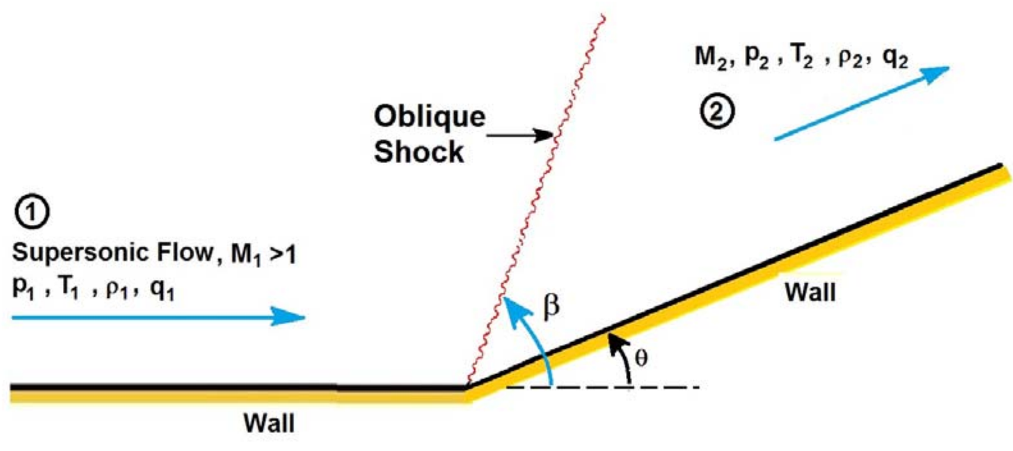

Figure 2: Supersonic flow performing a large compressive deflection, θ.

is non-isentropic.

We return to the basic conservation laws in order to construct (1) the inclination of the oblique shock wave, β, due to a finite deflection angle, θ, and (2) the relations between the flow properties ahead of and behind an oblique shock as a function of the upstream Mach number, M_1, and the angle, θ. To do so we will consider the components of the velocity normal to the oblique shock (denoted by q_N) and tangential to the shock (denoted by q_T) both upstream of the shock (subscript 1) and downstream (subscript 2) as depicted in Figure 3.

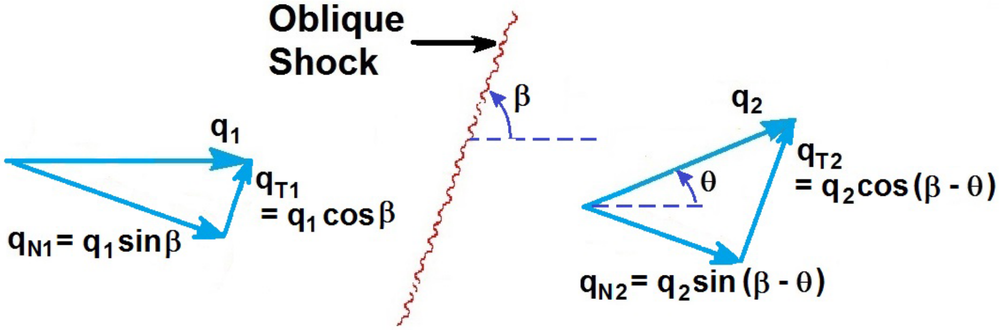

Figure 3: Notation for the oblique shock wave analysis.

- *Continuity*:
$$\rho_1 q_{N1} \;=\; \rho_2 q_{N2} \qquad \text{(Bom1)}$$
since the shock is assume infinitely thin and therefore the area of the flow is the same on both sides.

- *Momentum normal to the shock*:
$$p_1 + \rho_1 q_{N1}^2 \;=\; p_2 + \rho_1 q_{N2}^2 \qquad \text{(Bom2)}$$

- *Momentum tangential to the shock*:
$$\rho_1\, q_{N1}\, q_{T1} \;=\; \rho_2\, q_{N2}\, q_{T2} \qquad \text{so} \qquad q_{T1} \;=\; q_{T2} \qquad \text{(Bom3)}$$
using the continuity equation (Bom1).

- *Energy*:
$$c_p T_1 + \frac{q_{N1}^2}{2} + \frac{q_{T1}^2}{2} \;=\; c_p T_2 + \frac{q_{N2}^2}{2} + \frac{q_{T2}^2}{2} \qquad \text{so} \qquad c_p T_1 + \frac{q_{N1}^2}{2} \;=\; c_p T_2 + \frac{q_{N2}^2}{2} \qquad \text{(Bom4)}$$
using the continuity equation (Bom1).

- *State*:
$$\frac{p_1}{\rho_1 T_1} \;=\; \frac{p_2}{\rho_2 T_2} \qquad \text{(Bom4)}$$

- *and by definition*:
$$M_1^2 \;=\; \frac{q_1^2}{\gamma \mathcal{R} T_1} \qquad \text{and} \qquad M_2^2 \;=\; \frac{q_2^2}{\gamma \mathcal{R} T_2} \qquad \text{(Bom5)}$$

These relations are utilized as follows. First by eliminating all but θ, M_1 and β, the following equation emerges:
$$\tan\theta \;=\; \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{\{2 + (\gamma + \cos 2\beta M_1^2)\}} \qquad \text{(Bom6)}$$
Given θ, M_1 and γ this allows evaluation of the shock angle, β. The graph in Figure 4 was constructed using this equation (Bom6). Notice that for a given upstream Mach number, M_1, and a given deflection angle, θ, there are two values of the shock angle, β, that are possible solutions. We delay discussion on this until the other pertinent results are identified.

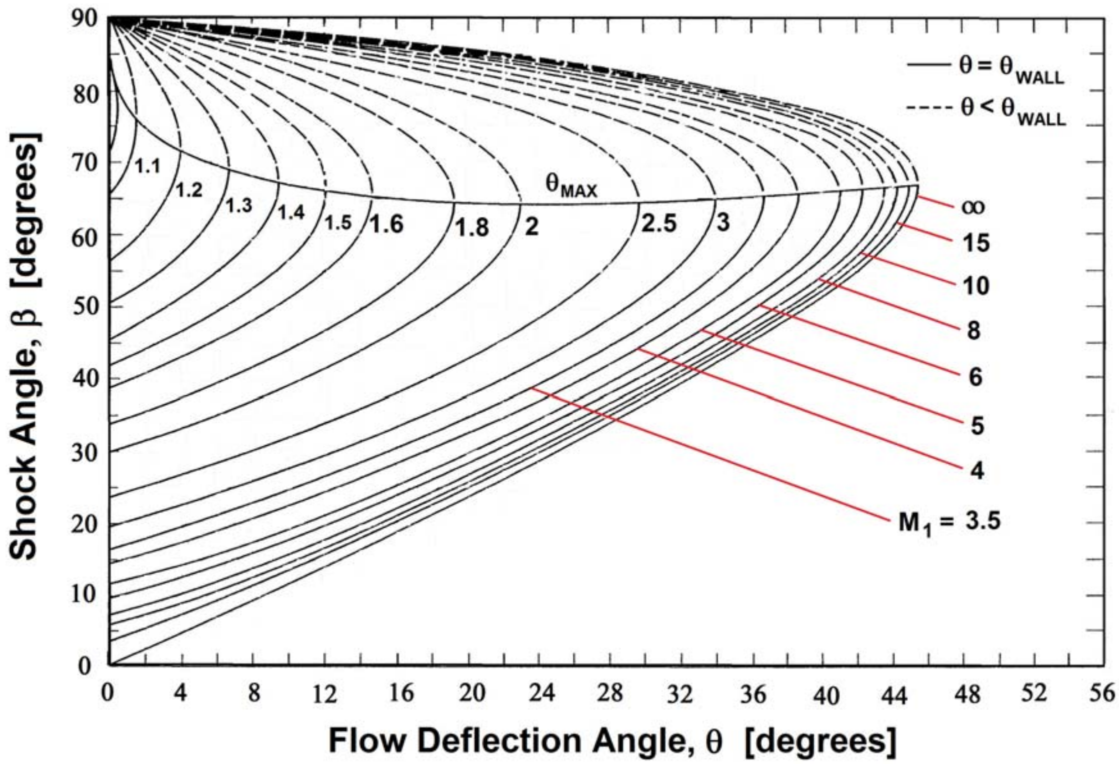

Figure 4: Data for oblique shock waves.

The second key result to emerge from the above set of equations is for the downstream Mach number, M_2, in terms of the upstream Mach number, M_1, θ, β, and γ:

$$M_2^2 \sin^2(\beta - \theta) = \frac{2 + (\gamma - 1)M_1^2 \sin^2\beta}{2\gamma M_1^2 \sin^2\beta - (\gamma - 1)} \quad \text{(Bom7)}$$

Careful examination of this relation shows that it is identical with the relation for M_2^2 for the normal shock (equation (Boh9)) except that

- $M_1 \sin\beta$ has replaced M_1 and
- $M_2 \sin(\beta - \theta)$ has replaced M_2

Therefore in order to calculate M_2 for an oblique shock the following steps should be followed

1. Obtain the shock angle, β, from the graph in Figure 4 using M_1 and the deflection angle, θ.
2. Calculate $M_1 \sin\beta$
3. Use the normal shock wave table in Figure 9 of section (Boh) with the column labeled M_1 relabeled $M_1 \sin\beta$ to look up the value of M_2 which is now interpreted as $M_2 \sin(\beta - \theta)$ and from this calculate M_2.
4. Use the same line of the table (Figure 9 of section (Boh)) to look up the pressure, temperature and density ratios across the oblique shock wave. Alternatively use

$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma - 1)(M_1^2 \sin^2\beta - 1)(\gamma M_1^2 \sin^2\beta + 1)}{(\gamma + 1)^2 M_1^2 \sin^2\beta} \quad \text{(Bom8)}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma(M_1^2 \sin^2\beta - 1)}{1 + \gamma} \quad \text{and} \quad \frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}\frac{p_2}{p_1} \quad \text{(Bom9)}$$

We note the following features of the oblique shock solution:

- As the shock angle, β, approaches $\pi/2$, the deflection angle, θ, tends to zero and the shock becomes a normal shock with $M_1 \sin\beta \rightarrow M_1$ and $M_2 \sin(\beta - \theta) \rightarrow M_2$.
- As the deflection angle, $\theta \rightarrow 0$, either (i) $\beta \rightarrow \pi/2$ which is the above limit or (ii) $M_1^2 \sin^2\beta \rightarrow 1$ and therefore $\beta \rightarrow \arcsin(1/M_1)$; in other words it becomes a Mach wave.
- In Figure 4, it is evident that for a given upstream Mach number, M_1, and a given deflection angle, θ, there are two values of the shock angle, β, that represent possible oblique shock configurations. The configurations in the lower part of Figure 4 shown by solid lines are characterized by downstream supersonic flows with $M_2 > 1$ and smaller shock angles, β. In contrast, the configurations in the upper part of Figure 4 shown by dashed lines are characterized by downstream subsonic flows with $M_2 < 1$ and larger shock angles, β. The dividing line between these two regimes is almost the locus of the vertical tangents in Figure 4 but lies just below it. The two shock types are known as a *weak oblique shock* when $M_2 > 1$ and a *strong oblique shock* when $M_2 < 1$. The pressure increase across a strong oblique shock is much larger than across a weak shock and hence the terminology. Whether a strong or weak shock occurs depends on the downstream conditions imposed on the flow. It is also important to recognize that the supersonic flow downstream of a weak shock must conform with the inclination of the wall downstream of the shock. On the other hand the subsonic flow downstream of a strong shock does not necessarily have to conform with the wall inclination. In fact the θ for a strong shock represents the inclination of the flow as it exits the shock and could be significantly smaller than the inclination of the wall. Consequently, completion of a strong shock solution requires the complex solution of the subsonic flow downstream of the shock.
- Note that for a given upstream Mach number, M_1, there is a maximum possible angle of deflection of the flow, θ_{max}. This is particularly evident when contemplating the solution for a wedge of half-angle, θ, placed symmetrically in a supersonic flow. When $\theta < \theta_{max}$ oblique shocks emanate from the vertex of the wedge. However, if $\theta > \theta_{max}$ the shock detaches from the vertex and becomes a "detached" shock wave placed some distance upstream of the vertex. Where it crosses the midline or line of symmetry it is, locally, a normal shock wave. Downstream of that detached shock the flow is subsonic and adjusts itself (and the position of the detached shock) to conform with the wedge geometry.

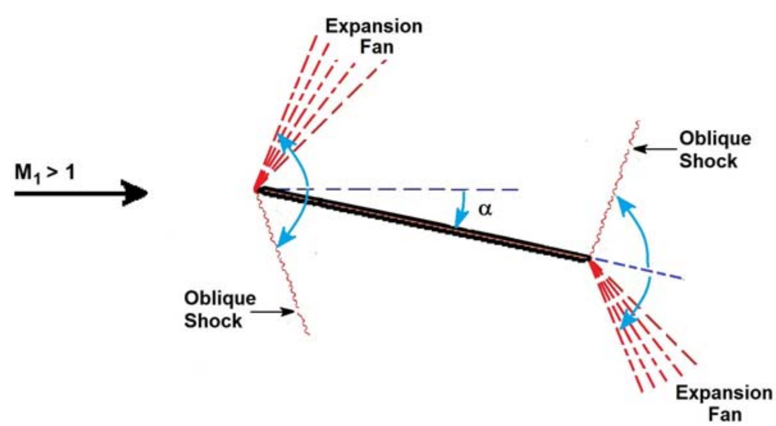

Figure 5: Supersonic flow past a flat plate at a substantial angle of attack.

- The large deflection angle versions of the various flows discussed in section (Boi) can now be constructed by substituting Prandtl-Meyer fans for the expansion Mach waves and oblique shock waves for the compression Mach waves. Thus, for example, the flow past a flat plate airfoil at a larger angle of attack becomes as depicted in Figure 5 and cases with given upstream Mach numbers and angles of attack can be analyzed for their lift and drag coefficients.

- The reflection of an oblique shock from a solid wall is somewhat similar to the reflection of a Mach wave delineated in section (Bok) except that the angle of reflection is not equal to the angle of incidence and needs to be calculated as in the following example depicted in Figure 6. Suppose that an oblique

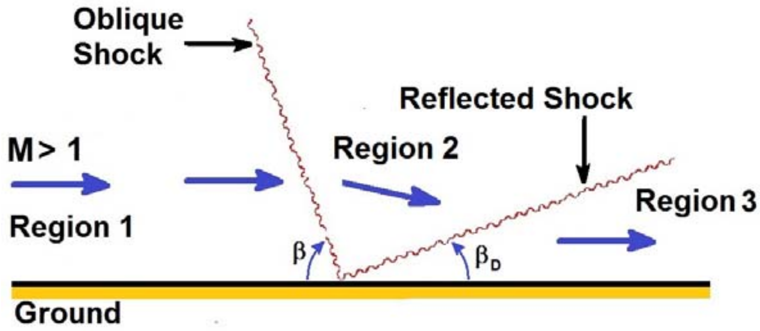

Figure 6: Oblique shock reflection.

incident shock with Mach number $M_1 = 3.0$ is inclined at an angle of $\beta_1 = 30^o$ to a wall parallel with the incident flow in region 1. With this input, the angle between the wall and the direction of the flow exiting the incident shock is 12.7^o according to Figure 4. This must also be the angle through which the flow is deflected in the reflected shock in order for the flow in region 3 to be parallel with the wall. Moreover, using the value of $M_1 \sin\beta_1 = 1.5$ and equation (Bom7) it transpires that $M_2 = 2.36$. Subsequently with $M_2 = 2.36$ and a deflection angle of 12.7^o, Figure 4 indicates that the inclination of the reflected shock to the direction of flow in region 2 must be 37^o. Hence the inclination of the reflected shock with the wall must be $(37 - 12.7) = 24.3^o$ and the Mach number in region 3 is 1.78. Thus the angle of the reflected shock relative to the wall (in this case 24.3^o) is smaller than the inclination of the incident shock (30^o); in contrast the angles were the same for a Mach wave.

- Another example of the "reflection" of an oblique shock wave occurs when oblique shock waves intersect. A useful example of such an interaction occurs in the processes that take place downstream of a over-expanded nozzle (Figure 7) as defined in section (Boi). As the external pressure is lowered below that at which a normal shock wave forms at the exit from the nozzle, that normal shock wave deforms outward into oblique shock waves emanating from the edge of the exit from the nozzle as shown in Figure 7. These oblique shock waves provide the mechanism by which the pressure increases to that downstream of this adjustment process. The intersection of the two oblique shock waves on

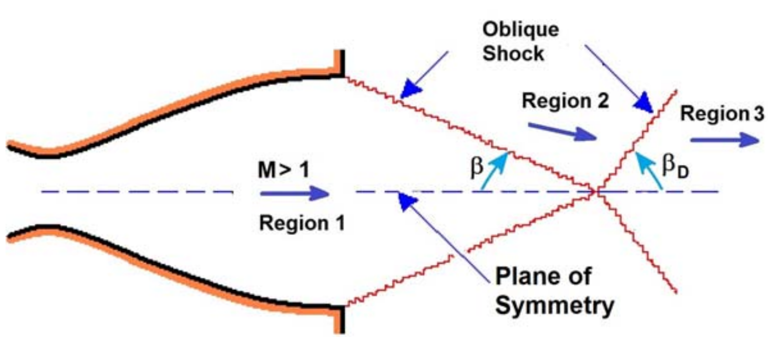

Figure 7: Oblique shock formation in the discharge from an over-expanded nozzle.

the centerline of nozzle is identical in form to the reflection of one of the oblique shocks if a solid wall were substituted for the plane of symmetry.