Prandtl-Meyer Expansion Fan

In this section we analyze supersonic flows with larger deflections, dealing first with expansion turns in which the change in inclination of the flow is away from the flow and is thus an *expansion deflection*. In the previous section the flow deflection was denoted by θ and defined as positive into the flow; however, in the present context of an expansion deflection, it will avoid confusion if we define a new variable, $\theta^* = -\theta$ so that θ^* is positive in this context. Then, examining the flow as sketched in Figure 1 it is clear that a gradual deflection will generate a set of Mach waves which diverge with distance from the surface. This is called an *expansion fan*, a *rarefaction fan* or a *Prandtl-Meyer fan*. Since it is comprised of Mach waves such a flow will be isentropic. Moreover, if we pan out from the perspective of Figure 1 as depicted in Figure 2 the expansion fan will be seen to emanate from the vertex and the flow will be isentropic. Consequently

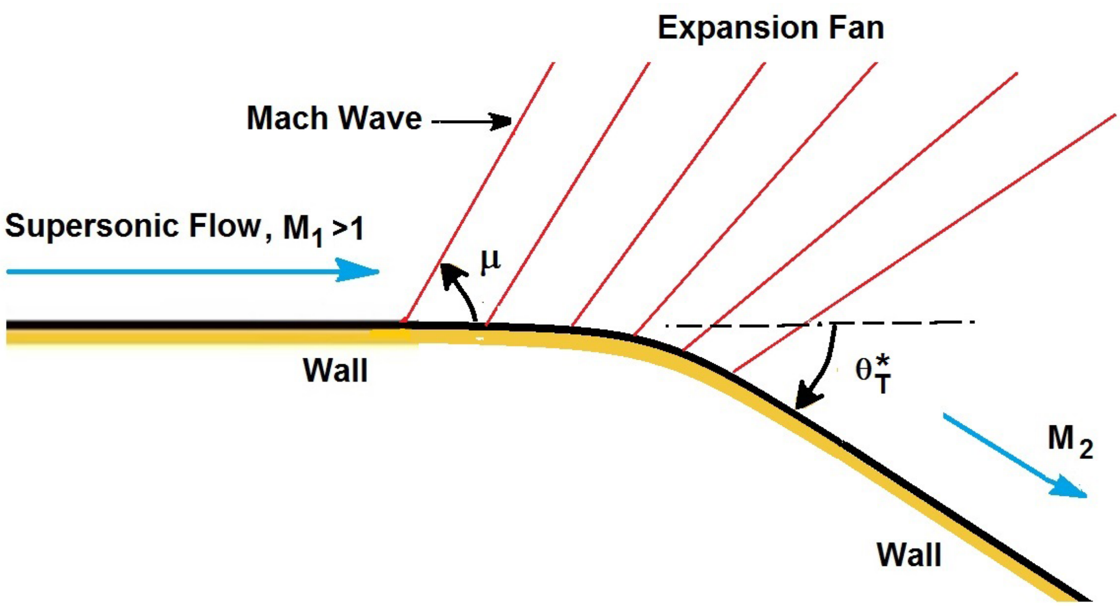

Figure 1: Supersonic flow performing a large, expansion deflection, θ^*_T.

the relation between the small changes in the inclination, $d\theta^*$, and the change in the Mach number, dM, will be given by equation (Bok9) or

$$d\theta^* = \frac{(M^2-1)^{\frac{1}{2}}}{\left\{1+\frac{(\gamma-1)}{2}M^2\right\}}\,\frac{dM}{M} \tag{Bol1}$$

and integrating this from the upstream conditions, $\theta^* = 0$, $M = M_1$, to the downstream conditions, $\theta^* = \theta^*_T$, $M = M_2$, yields

$$\theta^*_T = \int_{M_1}^{M_2} \frac{(M^2-1)^{\frac{1}{2}}}{\left\{1+\frac{(\gamma-1)}{2}M^2\right\}}\,\frac{dM}{M} = Pm(M_2) - Pm(M_1) \tag{Bol2}$$

where the special function, $Pm(M)$, is defined by

$$Pm(M) = \int_{1}^{M} \frac{(M^2-1)^{\frac{1}{2}}}{\left\{1+\frac{(\gamma-1)}{2}M^2\right\}}\,\frac{dM}{M} \tag{Bol3}$$

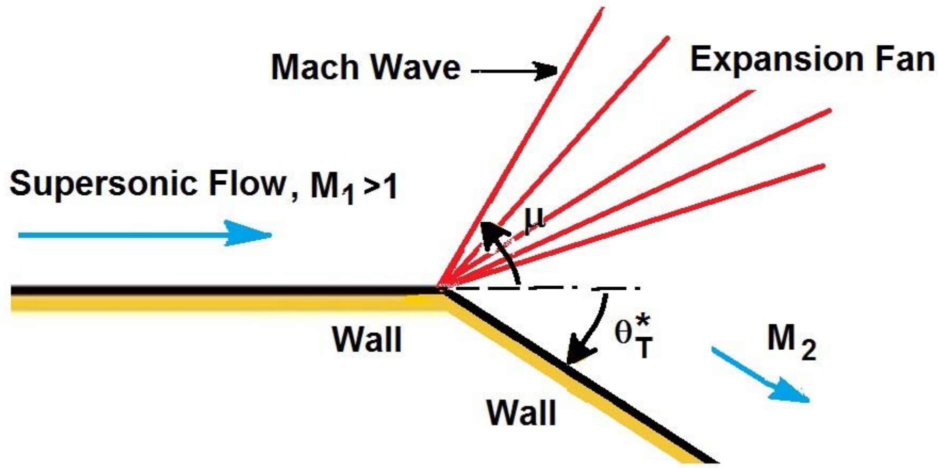

Figure 2: Supersonic flow performing a large, expansion deflection, θ_T^*.

and is known as the *Prandtl-Meyer function*. The graph of $Pm(M)$ is included as Figure 3 and is utilized in the following example. If the upstream Mach number is $M_1 = 4$ one first uses Figure 3 to ascertain that

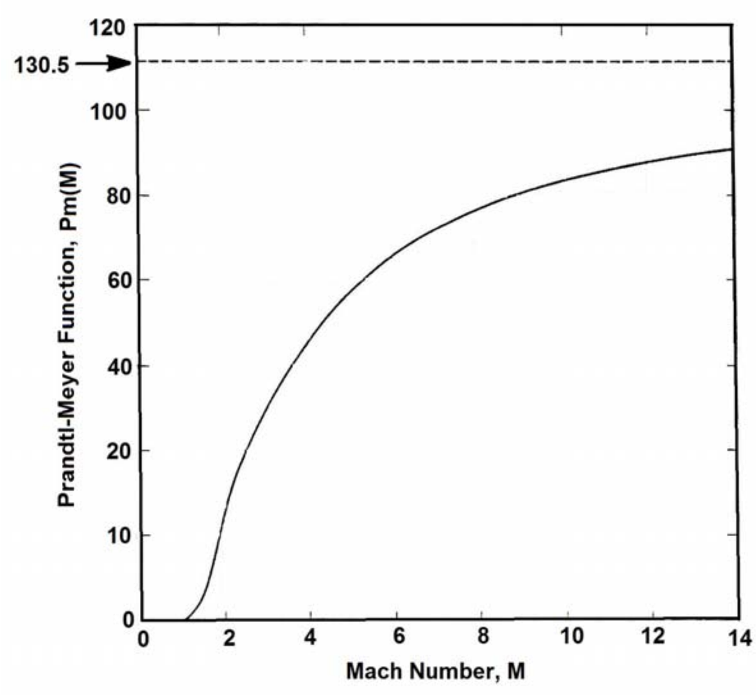

Figure 3: Prandtl-Meyer Function, $Pm(M)$, plotted against the Mach number, M.

$Pm(M_1) = 65^o$. Then, if the deflection angle $\theta_T^* = 30^o$, it follows from equation (Bol2) that $Pm(M_2) = 95^o$ and, using Figure 3 for a second time, that $M_2 \approx 8$. Moreover, since the process is isentropic, the tables and figures included in section (Boe) (Figures 1, 2 and 3 of section (Boe)) can then be used to find the relations between the pressures, temperatures and densities upstream ($M = M_1$) and downstream ($M = M_2$) of the expansion fan.

Note from Figure 3 that the Prandtl-Meyer Function, $Pm(M)$, has a maximum value of about 130.5 degrees. This implies that, for a given upstream Mach Number, M_1, there is a maximum angle that the flow can negotiate. For example, for $M_1 = 4$ and $Pm(M_1) = 65^o$, the maximum negotiable deflection is

65.5^o at which, in theory, $M_2 \rightarrow \infty$. In practice what happens is that the flow separates from the wall before it reaches this deflection.