Descriptions of Fluid Motion

In order to investigate fluid motion, one must first find an appropriate and useful way of represesenting that motion in mathematical terms. There are two important ways in which to approach this. One way would be to imagine fixing attention on a particular, small element of fluid (conceptually labelling a group of molecules) and to describe how the position, x_i, pressure, p, temperature, T, etc., of this element change with time, t:

$$x_i(t) \qquad ; \qquad p(t) \qquad ; \qquad T(t) \tag{Baa1}$$

where the velocity, $u_i(t)$, would then follow simply by differentiation of $x_i(t)$. This is known as the Lagrangian description of the motion. Of course, the various elements that one might choose could have quite different $x_i(t)$, $p(t)$, $T(t)$, etc. and therefore, in order to depict the whole fluid motion we would need to factor into this a unique label for each of the fluid elements. One possible choice for the unique label would be the position of the fluid element at some initial point in time which we could denote, say, by X_i. Then the whole motion is described by

$$x_i(X_i, t) \qquad ; \qquad p(X_i, t) \qquad ; \qquad T(X_i, t) \tag{Baa2}$$

This Lagrangian view of the motion has some advantages: for example, most of the fundamental laws of physics that we might wish to apply to the fluid flow, pertain to a particular mass of fluid, a mass made up of a particular group of molecules. However, it also has some disadvantages: for example, this is not a particularly convenient way to identify the flow conditions at a particular point on the surface of an aircraft.

The second important way in which one might describe the motion is to fix attention on a particular point in space, say a particular point in some Cartesian coordinate system, x_1, x_2, x_3 (or x_i) and to describe how the velocity, u_i, pressure, p, temperature, T, etc. vary with time at that and all other fixed points, x_i. Then the whole motion is described by

$$u_i(x_i, t) \qquad ; \qquad p(x_i, t) \qquad ; \qquad T(x_i, t) \tag{Baa3}$$

This is known as the Eulerian description of motion and is more satisfactory from an engineering or practical point of view because we can, for example, fix our coordinate system in the aircraft and thus readily identify the velocity of the flow at a particular point surrounding the aircraft.

Thus we see that there are two different ways to describe fluid motion mathematically, the Lagrangian and Eulerian descriptions. The Lagrangian description which focuses on a particular element of fluid is ideally suited for the application of the laws of Newtonian physics, conservation of mass, Newton's laws of motion and the basic laws of thermodynamics. Much of what we do in developing the equations of fluid motion is precisely this process of applying these laws to a Lagrangian fluid mass. However, the Lagrangian description is normally not very convenient from an engineering point of view since we most often wish to focus on a particular point in space rather than a particular fluid element. Hence the second step in the development of the equations for fluid motion involves converting the basic laws to an Eulerian frame of reference.

The first tool that we will need for this conversion process is a relation between the time derivatives within each of these descriptions of motion.