Illustration of Fluid Element Deformations

For the purposes of illustrating the decomposition of fluid motion detailed in the section on the kinematics of motion, it may be helpful to visualize the effects of the velocity gradients on a very small fluid element with dimensions that are large compared with the atomic and molecular dimensions but small compared with any macroscopic dimensions of the flow under consideration. For simplicity we will do this in two dimensions.

In a two-dimensional planar flow in the xy plane without any velocity in the z direction, the components of the strain rate tensor become

$$e_{xx} = \frac{\partial u}{\partial x} \quad ; \quad e_{yy} = \frac{\partial v}{\partial y} \tag{Bbb1}$$

$$e_{xy} = e_{yx} = \frac{1}{2}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\} \tag{Bbb2}$$

and, written out in terms of its Cartesian components, the only vorticity component, ω_z, which, for simplicity, is denoted by the scalar, ω, is

$$\omega = \omega_z = \left\{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right\} \tag{Bbb3}$$

We visualize a Lagrangian element, $ABCD$, with dimensions $dx \times dy \times dz$ as shown in figure 1 and consider the effects of the velocity gradients on the shape distortions of this element. Consider the dashed element

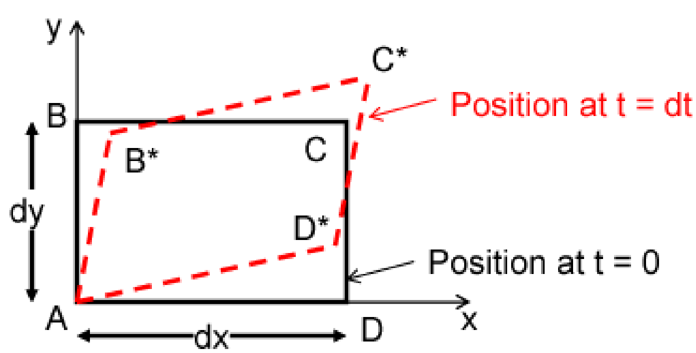

Figure 1: A small Lagrangian element of fluid.

$AB^*C^*D^*$ which represents the shape of the Lagrangian fluid element a short time, dt, later; we have removed the translation of the element by a Galilean transformation that superimposes the point A on its original location. Then, because of the velocity gradients in the flow the angle

$$B\hat{A}B^* = \frac{\partial u}{\partial y}\, dt \tag{Bbb4}$$

and the angle

$$D\hat{A}D^* = \frac{\partial v}{\partial x}\, dt \tag{Bbb5}$$

and therefore the deformation of the element is given by the difference between the angles

$$B^* \hat{A} D^* - B \hat{A} D = \frac{\partial u}{\partial y}\, dt + \frac{\partial v}{\partial x}\, dt - \frac{\pi}{2} \tag{Bbb6}$$

so the rate of deformation is

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{Bbb7}$$

in accord with the expression (Bbb2).

Moreover the rotation of the element is given by the angle between the diagonals AC and AC^* which, by simple geometry, is

$$\frac{1}{2}\left\{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right\} dt \tag{Bbb8}$$

so that the rate of rotation is

$$\frac{1}{2}\left\{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right\} \tag{Bbb9}$$

in accord with the expression (Bbb3).