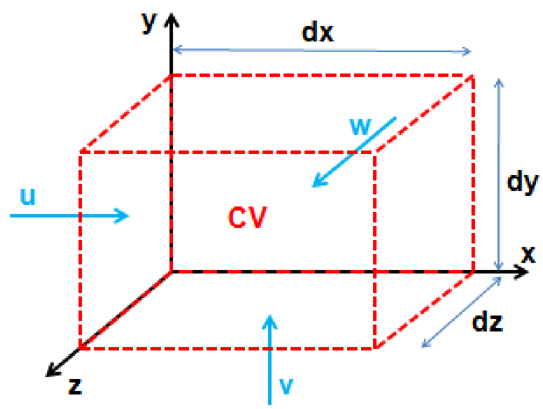
Control Volume

An important fundamental step in setting up the equations of fluid flow in any particular context is the establishment of a well-defined **control volume**. This is a volume which is usually fixed in some coordinate system (commonly an Eulerian volume) and whose entire surface is precisely defined. In places it may coincide with a fluid/solid interface and in other places it may cut across the flow. In a given problem or context it may include only fluid; in other problems or contexts it may include both fluid and structure. Often the most convenient and productive choice of control volume will depend on the desired outcome of the analysis applied to the control volume. Therefore several choices of control volume might be explored before the best choice becomes evident. In almost all cases the purpose is to apply some conservation principle to the control volume and therefore it will become neccessary to evaluate the flux of that conserved quantity into or out of the control volume through **all** of its surfaces. Consequently this evaluation needs to be borne in mind when defining the control volume.

A few examples could be useful. The infinitesmal control volume $dx \times dy \times dz$ shown in Figure 1(left) is commonly used in deriving the differential form of the equation that results from the application of a conservation principle to some fluid flow. In the present text this will be used in applying the principle of conservation of mass and Newton's law to quote two examples. On the other hand useful global results



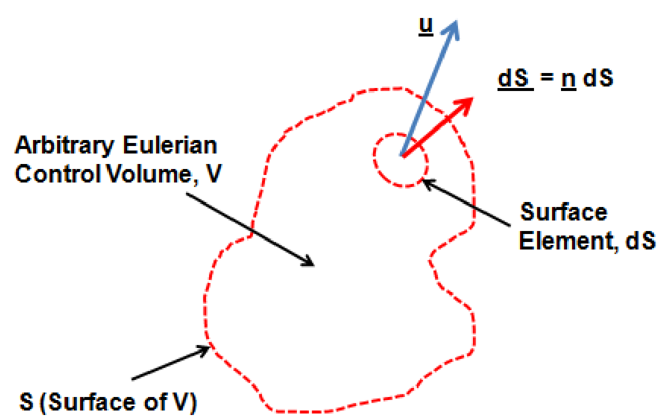


Figure 1: Infinitesmal (left) and Arbitrary (right) Eulerian control volumes (dashed red lines).

can often emerge when the conservation principle is applied to a large, macroscopic control volume such as that depicted in Figure 1(right). The results that emerge from such an analysis usually take the form of an integral equation rather than a differential equation. We shall also use this in developing integral forms for the conservation of mass and Newton's law as well as the second law of thermodynamics (to quote three examples).

It is often the case that a judicious choice of a macroscopic control volume in a particular engineering problem can yield very useful results. If, for example, a conservation principle is to be applied to the tank with three connecting pipes shown on the left in Figure 2 then one possible choice of the control volume is indicated by the surface defined by the dashed red lines. Another possible choice would be the dashed blue line shown on the right in Figure 2. We shall explore these kinds of alternatives in other sections of this book, for example during our discussions of the thrust produced by jet engines and rocket engines.

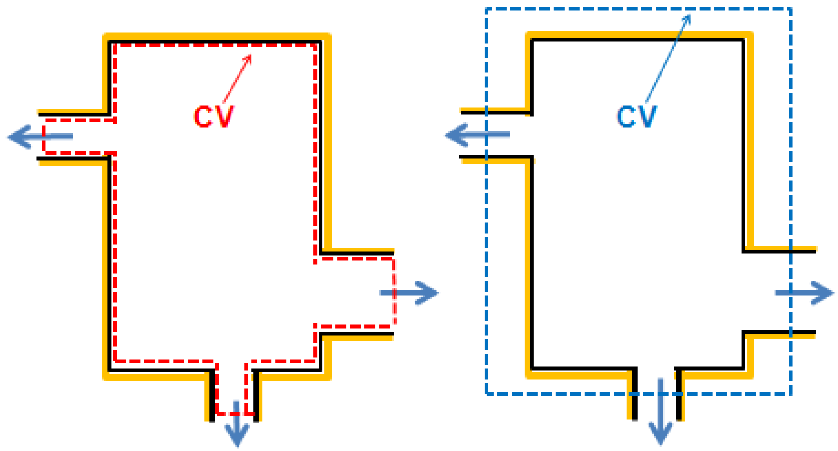


Figure 2: Two alternative Eulerian control volumes.