Example: Fire Hose Nozzle

As a common and useful example consider the application of the linear momentun theorem to a simple nozzle as shown in Figure 1. For simplicity, it is assumed that the velocity magnitudes, q_1 and q_2, are uniform across the inlet area, A_1, and the jet area, A_2, respectively. The fluid is incompressible with density ρ and viscous losses within the nozzle are negligible as are the effects of gravity.
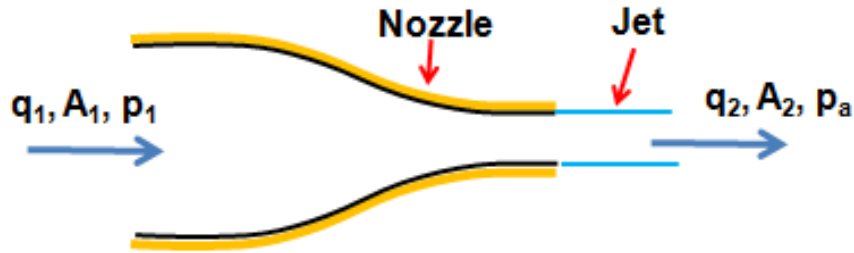

Figure 1: A simple nozzle.

It follows from the arbitrary duct analysis that the force that must be applied to the nozzle to hold it in place is

$$F_1^* = m(-q_1 + q_2) - (p_1 - p_a)A_1 + (p_2 - p_a)A_2 \qquad \text{(Bec1)}$$

in the direction of the flow. In this case the jet pressure, p_2, is equal to the atmospheric pressure, p_a, and the mass flow rate, $m = \rho q_1 A_1 = \rho q_2 A_2$, by continuity and so this expression can be written as

$$F_1^* = \rho A_1 q_1^2 \left\{ \frac{A_1}{A_2} - 1 \right\} - (p_1 - p_a)A_1 \qquad \text{(Bec2)}$$

Moreover, since the viscous losses in the nozzle are deemed negligible and gravity is to be neglected we can use Bernoulli's to relate the pressure difference to the velocities:

$$p_1 - p_a = \frac{1}{2}\rho(q_2^2 - q_1^2) = \frac{1}{2}\rho q_1^2 \left\{ \frac{A_1^2}{A_2^2} - 1 \right\} \qquad \text{(Bec3)}$$

and so the force that one must apply to the nozzle to hold it in place becomes

$$F_1^* = -\frac{1}{2}\rho A_1 q_1^2 \left\{ \frac{A_1}{A_2} - 1 \right\}^2 \qquad \text{(Bec4)}$$

or, in terms of the pressure difference, $(p_1 - p_a)$,

$$F_1^* = -(p_1 - p_a)A_1 \frac{(A_1 - A_2)}{(A_1 + A_2)} \qquad \text{(Bec5)}$$

In fact this is the force that the pipe upstream of the nozzle applies to the nozzle. The fact that this force is in the backward direction (opposite to the direction of flow) is somewhat counterintuitive since momentum is being added to the fluid as it passes through the nozzle. But we note that the total force being applied to the fluid in the nozzle is F_1^* plus $(p_1 - p_a)A_1$ or

$$F_1 = F_1^* + (p_1 - p_a)A_1 = (p_1 - p_a)A_1 \frac{2A_2}{(A_1 + A_2)} \qquad \text{(Bec6)}$$

which is in the forward direction. A fireman holding a nozzle with a flexible hose must indeed apply this forward force while holding the nozzle in order to prevent it flying backwards. This is seen more clearly in the next example of a fire hose with an upstream bend.