Rocket Engine Thrust

The thrust produced by a rocket engine is most readily understood through the application of the linear momentum theorem. Consider the sketch of the cross-section of a rocket engine as shown in the figure below. The outline of the rocket engine discharge is shown by the dashed blue lines. The far upstream

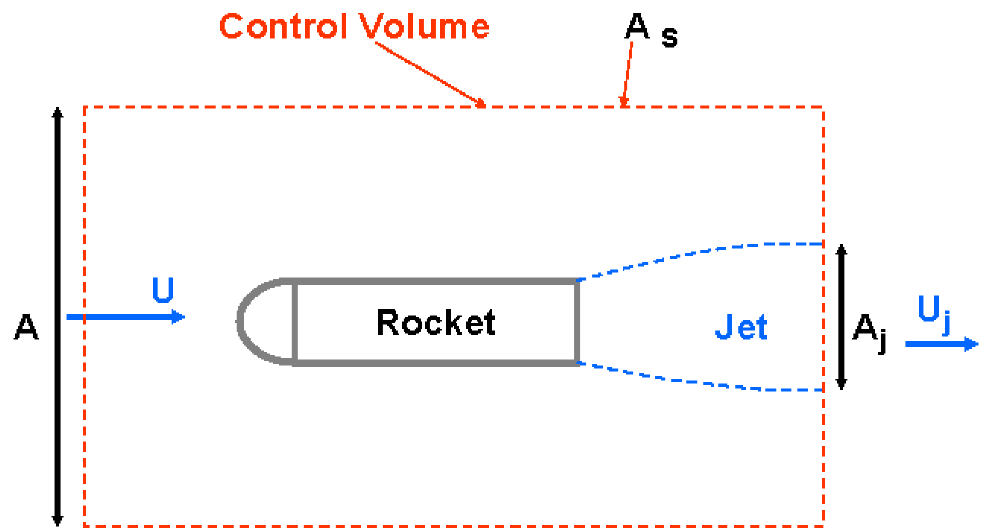

Figure 1: A cross-section through a rocket engine showing the outline of the rocket discharge (dashed blue lines) and a cylindrical control volume (dashed red lines).

velocity of the oncoming stream and its density are denoted by U and ρ. Far downstream the cross-section of the jet emerging from the engine is denoted by A_j. The velocity and density of this jet is denoted by U_j and ρ_j. We assume that any mixing between the jet and the surrounding fluid can be neglected so that the bounding streamtube surface represents a discontinuity in velocity. Viscous effects in the exterior flow are also neglected so that the velocity of the fluid exterior to the jet is given by U.

We define a large cylindrical control volume as shown by the dashed red lines. The various components of the surface of this control volume are:

- A large upstream area, A_F, normal to the oncoming stream which is sufficiently far from the engine so that the pressure on that surface is essentially the atmospheric pressure far upstream.
- A large cylindrical surface, A_S, which is everywhere parallel with U that represents the outer boundary of the control volume.
- A downstream surface normal to the oncoming stream which is the other end of the cylindrical control volume and therefore also has an area A_F.
- Within this downstream area is the intersection of the jet which has an area A_j.

Therefore, except for the jet area, all the flows on the boundaries of this control volume have a velocity in the U direction equal to U and a density of ρ. In contrast, the rocket exhaust jet has a velocity U_j (which, for simplicity, we will assume is uniform across the jet) and a density ρ_j.

With these definitions we can now apply conservation of mass and then the momentum theorem in the U direction. Assuming that the flow is steady (for simplicity we will neglect the fact that the rocket may be accelerating) continuity of the fluid flow exterior to the jet requires that

$$\rho U A = \rho U (A - A_j) + \dot{M} \tag{Beg1}$$

where $\dot{M}$ denotes the mass flow out through the sides of the control volume, namely the area A_S. It follows that

$$\dot{M} = \rho U A_j \tag{Beg2}$$

In addition the mass flux in the exhaust jet must equal the rate of decrease of the mass, $\mathcal{M}$, of the rocket so that

$$\frac{d\mathcal{M}}{dt} = -\rho_j U_j A_j \tag{Beg3}$$

Now we apply the momentum thereom in the U direction to obtain the total force F acting on the contents of the control volume (which includes the rocket engine and the jet) in the U direction:

$$F = -\rho U^2 A + \rho U^2 (A - A_j) + \rho_j U_j^2 A_j + U\dot{M} \tag{Beg4}$$

and, with cancellations and the substitution for $\dot{M}$ from the expression derived from continuity, this becomes

$$F = \rho_j U_j^2 A_j \tag{Beg5}$$

Finally we must consider the various possible contributions to the total force, F, acting on the control volume and its contents in the U direction. It is assumed that the flow has a sufficiently high Reynolds number so that the shear stresses acting on A_0 are negligible and so that there are no significant viscous contributions to the normal stresses on the surfaces normal to U. Thus the only pertinent forces acting on the external surface of the control volume are those due to the pressure. Moreover it is assumed that these surfaces are sufficiently far from the body that the pressures on all surfaces are equal to the pressure in the uniform stream. It follows that there is no contribution of the pressures to F. Consequently if we neglect contributions from body forces such as gravity (or assume U is horizontal), the only contribution to F is the force that would have to be applied to the body to hold it in place. In the simple case in which we neglect the acceleration of the rocket, that force will be the thrust produced by the rocket, T. It follows that the thrust, T, produced by the engine in the *positive* U direction is

$$T = \rho_j U_j^2 A_j \tag{Beg6}$$

Note how the thrust is simply the mass flow rate in the exhaust, $\dot{M}_j = \rho_j U_j A_j$, multiplied by the jet velocity, U_j.

In this simplified analysis we have assumed that the velocity, U_j, is uniform across the jet area, A_j. As in the case of the jet engine non-uniformity increases the thrust for a given flow rate, $\dot{M}_j$. Also, in this simplified analysis we have neglected the effect of the acceleration of the rocket which needs to be incorporated in a more complete analysis.