Velocity Potential

Irrotational flow is defined as a flow in which the vorticity, $\underline{\omega}$, is zero and since

$$\underline{\omega} = \nabla \times \underline{u} \tag{Bdf1}$$

it follows that the condition, $\underline{\omega} = 0$, is automatically satisfied by defining a quantity called the **velocity potential**, ϕ, such that

$$\underline{u} = \nabla \phi \tag{Bdf2}$$

since it is always true that $\nabla \times \nabla \phi = 0$. For this reason irrotational flow is often called **potential flow** and we will refer to it as such. Other forms of equation (Bdf2) are

$$u_i = \frac{\partial \phi}{\partial x_i} \quad ; \quad u = \frac{\partial \phi}{\partial x}\, ,\, v = \frac{\partial \phi}{\partial y}\, ,\, w = \frac{\partial \phi}{\partial z} \tag{Bdf3}$$

It is important not to confuse the velocity potential, ϕ, with the streamfunction, ψ. The former can be defined in any three-dimensional flow whereas a streamfunction can only be defined in some two-dimensional flows. For example for incompressible planar flow

$$u = \frac{\partial \phi}{\partial x} \quad , \quad v = \frac{\partial \phi}{\partial y} \;\; \text{but} \;\; u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \tag{Bdf4}$$

These are called the Cauchy-Riemann equations and they imply that, in incompressible planar potential flow, the lines of constant velocity potential (called **equipotentials**) are everywhere perpendicular to the lines of constant ψ, namely streamlines. Therefore, in sketches of incompressible planar potential flow these lines, streamlines and equipotentials, form an orthogonal net that can be visualized as covering the flow as shown by example in Figure 1.

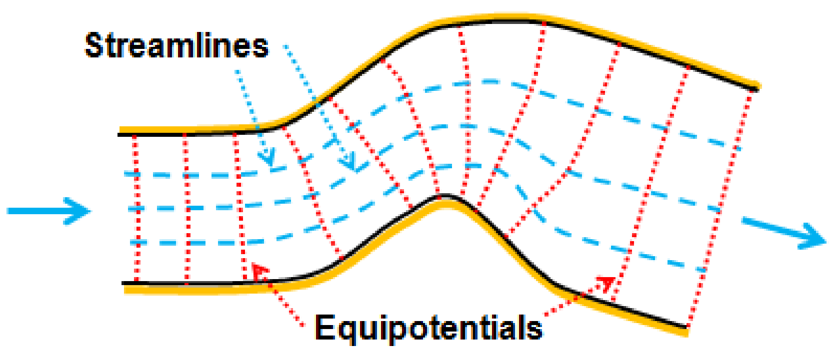

Figure 1: The orthogonal flow net of streamlines and equipotentials in an incompressible planar potential flow.

Another key feature of irrotational flow is Bernoulli's equation to which the reader should now turn.