The Effect of Friction

In many open channel flow analyses, it is necessary to include the effect of friction at the channel base or sides. To illustrate the effect of a non-zero shear stress, τ_w, at the base or side consider the simple open channel flow down an inclined plane as depicted in Figure 1. We apply the continuity equation and the

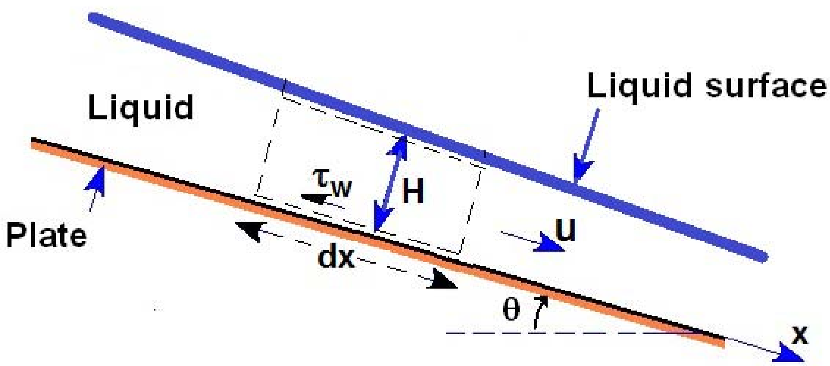

Figure 1: Open channel flow down an inclined plane with friction, τ_w.

linear momentum theorem in the x-direction to the infinitesmal element dx that spans the entire depth, H, of the layer. The continuity equation requires that

$$\frac{d}{dx}(uH) = 0 \quad \text{and} \quad u\frac{dH}{dx} = -H\frac{du}{dx} \tag{Bpe1}$$

The linear momentum theorem in the x-direction yields

$$\rho g H \sin\theta dx - \rho g H \frac{dH}{dx}dx - \tau_w dx = \frac{d}{dx}\left(\rho H u^2\right) dx \tag{Bpe2}$$

and using equation (Bpe1)

$$(gH - u^2)\frac{dH}{dx} = gH\sin\theta - \frac{\tau_w}{\rho} \tag{Bpe3}$$

Recalling that the *friction coefficient*, f, is $f = 8\tau_w/\rho u^2$ this can be written as

$$(1 - Fr^2)\frac{dH}{dx} = \sin\theta - \frac{fFr^2}{8} \tag{Bpe4}$$

This equation manifests the same kind of frictional effects that were described in the context of compressible flows. Specifically

- When $\sin\theta = \tau_w/\rho g H$ it follows that the flow is neither accelerating not decelerating and $dH/dx = 0$; this defines a critical bed slope, θ_c, whose observation allows a practical estimate of τ_w or the friction factor, f.
- In a subcritical flow ($Fr < 1$) when $\sin\theta < fFr^2/8$ then $dH/dx < 0$, the depth decreases with distance x and the Froude number, Fr, increases. Consequently, provided $\sin\theta < fFr^2/8$ a subcritical flow must inevitably tend to a critical limit, $Fr = 1$.

- On the other hand in a supercritical flow ($Fr > 1$) when $\sin\theta < fFr^2/8$ then $dH/dx > 0$, the depth increases with distance x and the Froude number, Fr, decreases. Consequently, provided $\sin\theta < fFr^2/8$ a supercritical flow must inevitably tend to a critical limit, $Fr = 1$.

Therefore, it follows that as long as $\sin\theta < f/8$ the Froude number must approach unity. This is termed a *controlled flume flow* since it is self limiting provided the slope of the conduit is less than the critical value of $\theta = \arcsin f/8$. In contrast, when the slope, θ, is greater than this critical value the flow will accelerate continually.

Clearly, the friction factor, $f = 8\tau_w/\rho u^2$, in a river or open channel will be a function not only of the cross-sectional geometry of the channel and the flow but also of the roughness of the surfaces in contact with the fluid. There are a number of empirical formula that are typically used to evaluate τ_w or f, most usually based on the friction factors used in turbulent flow described in sections (Bk). For example in a river with a sufficiently rough bed for the flow to be *fully rough turbulent flow*, the friction factor will be primarily a function of ϵ/H where ϵ is the typical roughness size, say $f = F(\epsilon/H)$. As mentioned above, the value of f is most commonly estimated by observing the slope, θ_c, at which the flow is neither accelerating or decelerating. One commonly used empirical formula relating the friction (specifically θ_c) to the volumetric flow velocity, u, and the typical dimension of the flow cross-section is Manning's formula which can be written as

$$\theta_c \text{ (in radians)} \;=\; n^2u^2/R^{4/3} \tag{Bpe5}$$

where R is the hydraulic radius of the cross-section of the flow and n is Manning's coefficient which is not dimensionless. It follows from the above relations that

$$n^2 \;\propto\; \frac{H^{1/3}}{g} f \tag{Bpe6}$$

so that, for fully rough turbulent flow

$$n^2 \;\propto\; \frac{H^{1/3}}{g} F(\epsilon/H) \tag{Bpe6}$$

Hydraulic engineers use tabulated values for n for different surface roughness elements to estimate θ_c and the friction factor, f.