The Shallow Water Wave Equations

Unsteady open-channnel flows are often treated using the shallow water wave equations that are pertinent to flows in which the wavelength of the waves is large compared with the depth, H. The development of the shallow water waves for planar flow utilizes a derivation similar to that described in section (Bpe) except that the unsteady terms in the continuity and momentum equations must be included. Referring

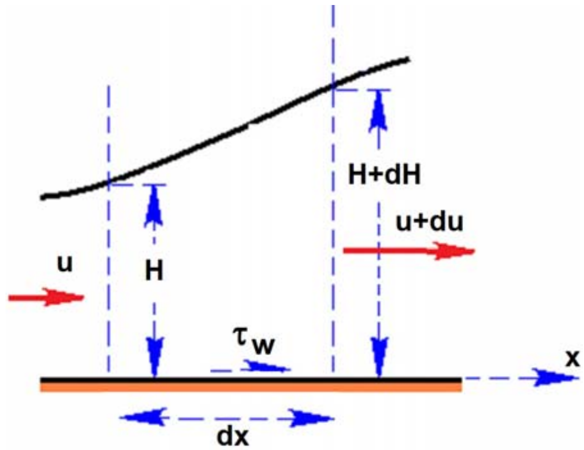

Figure 1: Fluid element.

to the control volume of length dx sketched in Figure 1 which spans the entire depth, H, of the layer, the unsteady continuity equation requires that

$$uH - \left\{ uH + \frac{\partial(uH)}{\partial x} dx \right\} = \frac{\partial H}{\partial t} dx \quad \text{(Bpf1)}$$

or

$$\frac{\partial H}{\partial t} + \frac{\partial(uH)}{\partial x} = 0 \quad \text{(Bpf2)}$$

The linear momentum thereom in the x-direction yields

$$\frac{\partial}{\partial t}(\rho H dx u) + \frac{\partial}{\partial x}\left(\rho H u^2\right) dx = -\rho g H \frac{\partial H}{\partial x} dx - \tau_w dx \quad \text{(Bpf3)}$$

where the first term is the rate of increase of x-momentum within the control volume, the second term is the net flux of x-momentum out of the control volume, the third is the net force due to the hydrostatic forces and the fourth is the friction force at the bottom. Using equation (Bpf2):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial H}{\partial x} - \frac{\tau_w}{\rho H} \quad \text{(Bpf4)}$$

Equations (Bpf2) and (Bpf4) constitute the planar, shallow water wave equations. This set of two, non-linear, partial differential equations need to be solved for the unknown functions, $u(x,t)$ and $H(x,t)$ given appropriate initial and/or boundary conditions. Commonly, the method of characteristics is used to solve these equations numerically.