Standing Gravity Waves

In the preceding sections we investigated traveling waves on an infinitely deep ocean and traveling waves on an ocean of finite depth. Here we note that in both cases, the solution to standing waves is easily obtained by simply adding the potential flow solution for waves traveling in the negative x direction to the solution for waves traveling in the positive x direction. This is most effectively done by adding the solution expressions in the preceding sections for waves traveling in the positive x direction to the same expressions but with ω replaced by $-\omega$. Thus the potential flow solution for standing waves on an infinitely deep ocean is

$$h(x,t) \;=\; h_M \sin{(kx-\omega t)} + h_M \sin{(kx+\omega t)} \;=\; 2h_M \sin{kx} \cos{\omega t} \tag{Bgce1}$$

$$\phi \;=\; -\frac{h_M\omega}{k}\cos{(kx-\omega t)}e^{ky} + \frac{h_M\omega}{k}\cos{(kx+\omega t)}e^{ky} \tag{Bgce2}$$

$$u \;=\; \frac{\partial\phi}{\partial x} \;=\; +h_M\omega \sin{(kx-\omega t)}e^{ky} - h_M\omega \sin{(kx+\omega t)}e^{ky} \tag{Bgce3}$$

$$v \;=\; \frac{\partial\phi}{\partial y} \;=\; -h_M\omega \cos{(kx-\omega t)}e^{ky} + h_M\omega \cos{(kx+\omega t)}e^{ky} \tag{Bgce4}$$

or

$$\phi \;=\; -\frac{2h_M\omega}{k}\;\sin{kx}\;\;\sin{\omega t}\;\;e^{ky} \tag{Bgce5}$$

$$u \;=\; \frac{\partial\phi}{\partial x} \;=\; -2h_M\omega\;\;\cos{kx}\;\;\sin{\omega t}\;\;e^{ky} \tag{Bgce6}$$

$$v \;=\; \frac{\partial\phi}{\partial y} \;=\; -2h_M\omega\;\;\sin{kx}\;\;\sin{\omega t}\;\;e^{ky} \tag{Bgce7}$$

Standing waves are illustrated in Figure 1, their amplitude oscillating in time while their location remains fixed. Note that there are nodes at $kx = 0$, π, 2π, etc. at which the surface elevation is always zero. For

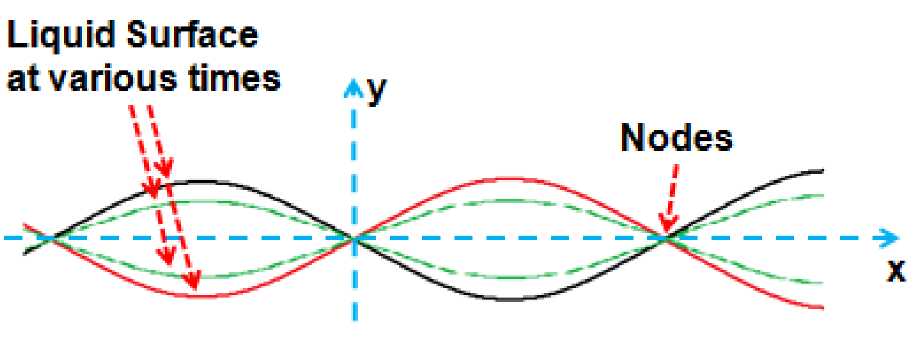

Figure 1: Illustration of standing waves.

future purposes, we take particular note of the fact that there is also a set of fixed x locations at which the horizontal velocity u is zero at all depths and all times, namely when $kx = \pi/2$, $3\pi/2$, etc. This feature is unique to standing waves; there is no such x location for traveling waves.

Also for future purposes we note that the potential flow solution for standing waves on an ocean of finite depth, H, can be constructed in precisely the same way and, for the same surface shape,

$$h(x,t) \;=\; 2h_M\;\;\sin{kx}\;\;\cos{\omega t} \tag{Bgce8}$$

the solution becomes

$$\phi = -\frac{2h_M\omega}{k\sinh kH}\ \sin kx\ \cosh k(y+H)\ \sin\omega t \tag{Bgce9}$$

$$u = \frac{\partial\phi}{\partial x} = -\frac{2h_M\omega}{\sinh kH}\ \cos kx\ \cosh k(y+H)\ \sin\omega t \tag{Bgce10}$$

$$v = \frac{\partial\phi}{\partial y} = -\frac{2h_M\omega}{\sinh kH}\ \sin kx\ \sinh k(y+H)\ \sin\omega t \tag{Bgce11}$$

Note that the x locations of the nodes and positions of zero horizontal velocity are the same as in the infinite depth case.