Methods of Solution of Laplace's Equation

We have noted that the solution of incompressible, inviscid, irrotational flows requires the solution of Laplace's equation, either

$$\nabla^2\phi = 0 \quad \text{or} \quad \nabla^2\psi = 0 \tag{Bgb1}$$

Various procedures for the solution of Laplace's equation are illustrated in the examples of potential flows that are laid out in the pages that follow. The following is a brief but not exhaustive list of the possible methodologies:

- The method of separation of variables in Cartesian coordinates. This is exemplified by the solutions for waves on an ocean surface.
- The method of separation of variables in polar coordinates. This is exemplified by the solutions for sources, sinks, doublets, point vortices, etc.
- Numerical methods. These are illustrated by a simple finite difference approach.
- Methods of complex variables. These are pertinent to planar flows and are illustrated by Jowkowski airfoil solutions.

A key facility in constructing a wide variety of solutions to incompressible, inviscid, irrotational flows is the strategy of superposition. Note that if ϕ_1 is a solution to Laplaces's equation and ϕ_2 is a second solution then since the equation (Bgb1) is linear in ϕ it follows that $\phi_1 + \phi_2$ is also a solution to Laplace's equation. Thus we may construct increasingly complicated solutions simply by adding together two or more simpler solutions. We shall use this strategy many times in the pages which follow. Of course, we need to ensure that the final product satisfies the boundary conditions that we need to impose.