Potential Flow around a Cylinder

Superimposing a uniform stream of velocity, U, on the potential flow due a doublet oriented in the x

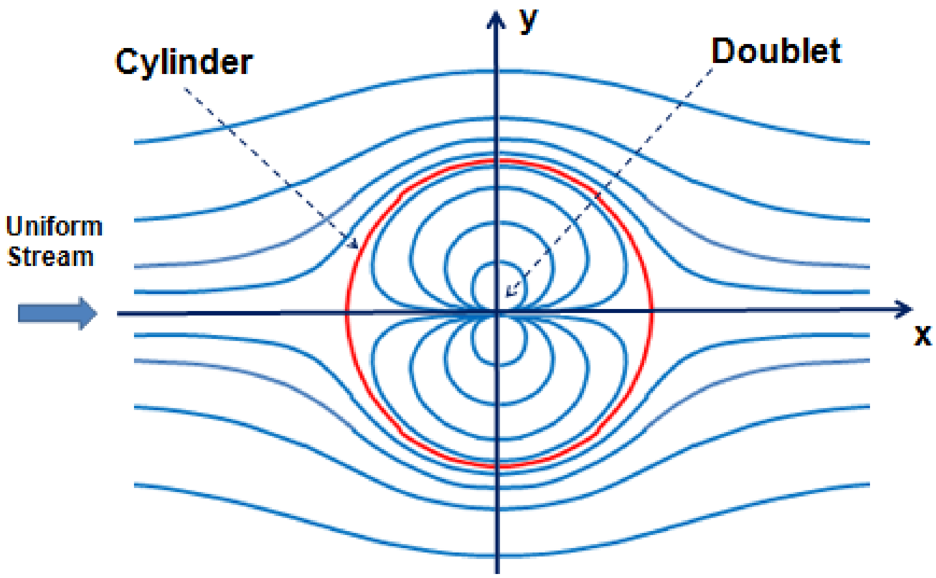

Figure 1: Streamlines in the potential flow of a doublet in a uniform stream.

direction produces the flow and streamlines shown in Figure 1 which has the velocity potential,

$$\phi = Ux + \frac{Qa}{\pi}\frac{x}{(x^2+y^2)} = \left[Ur + \frac{Qa}{\pi r}\right]\cos\theta \tag{Bgdh1}$$

and the radial velocity

$$u_r = \frac{\partial\phi}{\partial r} = \left[U - \frac{Qa}{\pi r^2}\right]\cos\theta \tag{Bgdh2}$$

where the polar coordinates, r and θ, are $x = r\cos\theta$ and $y = r\sin\theta$ as before. It follows that at the specific radius, $r = R = (Qa/\pi U)^{\frac{1}{2}}$ the radial velocity is zero for all θ. Therefore the radius, $r = R$, is a streamline and could, if so desired, be replaced by a cylinder of that radius in order to generate the potential flow around a cylinder (shown by the red circle in Figure 1). It follows that the potential flow around a cylinder of radius R is characterized by the velocity potential, velocity components and stream function given by

$$\phi = \left[r + \frac{R^2}{r}\right]U\cos\theta \tag{Bgdh3}$$

$$u_r = \frac{\partial\phi}{\partial r} = \left[1 - \frac{R^2}{r^2}\right]U\cos\theta \tag{Bgdh4}$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\left[1 + \frac{R^2}{r^2}\right]U\sin\theta \tag{Bgdh5}$$

$$\psi = \left[r - \frac{R^2}{r}\right]U\sin\theta \tag{Bgdh6}$$

It will be useful for future purposes to investigate some of the properties of this flow. First note that the tangential velocity on the surface of the cylinder is given by

$$(u_\theta)_{r=R} \;=\; -2U\sin\theta \tag{Bgdh7}$$

As expected the velocity on the surface increases from zero at the front stagnation point ($\theta = \pi$) to a maximum of $2U$ at the "equator" ($\theta = \pi/2$) and then decreases again to zero at the rear stagnation point ($\theta = 0$). By Bernoulli's theorem it follows that, neglecting gravity, the pressure, p, on the surface of the cylinder is given by

$$(p)_{r=R} \;=\; p_\infty + \frac{1}{2}\rho U^2 - \frac{1}{2}\{2U\sin\theta\}^2 \;=\; p_\infty + \frac{1}{2}\rho U^2\left\{1 - 4\sin^2\theta\right\} \tag{Bgdh8}$$

where p_∞ is the pressure far upstream. It is conventional to define a non-dimensional **coefficient of pressure** denoted by C_p as

$$C_p \;=\; \frac{p - p_\infty}{\frac{1}{2}\rho U^2} \tag{Bgdh9}$$

and it follows from equation (Bgdh8) that the coefficient of pressure on the surface of the cylinder is given by

$$(C_p)_{r=R} \;=\; 1 - 4\sin^2\theta \tag{Bgdh10}$$

Since the pressure on the surface of the cylinder, $(p)_{r=R}$, is symmetric fore and aft it must follow that the drag on the cylinder in potential flow is identically zero. This is, again, an example of D'Alembert's Paradox which states that the drag on any finite body due to potential flow must be zero. We shall revisit this issue in future pages and resolve the apparent conflict with our practical experience. For the present we take note of the sinusoidal pressure distribution on the surface cylinder depicted in Figure 2. Note

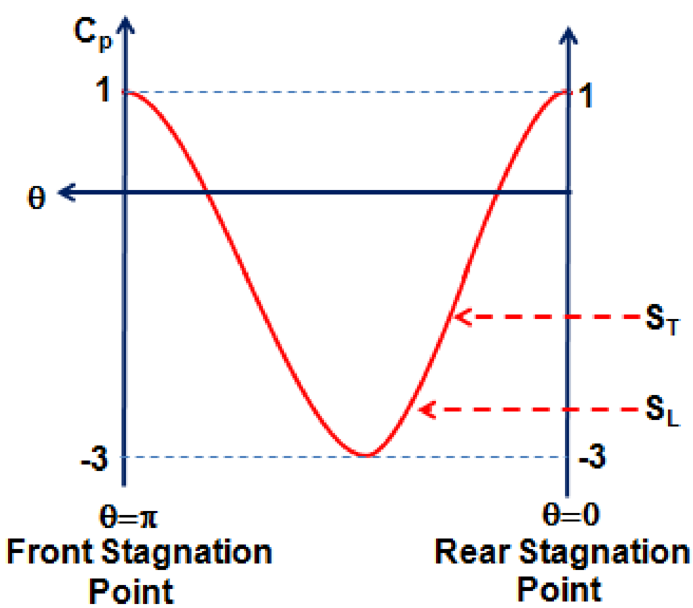

Figure 2: Pressure distribution on the surface of a cylinder in the potential flow.

that C_p at the front and rear stagnation points is unity as it always is where the velocity is zero. Note the minimum pressure coefficient of -3 at the equator, $\theta = \pi/2$ and the symmetry of the potential flow pressure distribution that leads to zero drag. Though we jump ahead, we should mention that in the actual flow around a cylinder the pressure over the front between $\theta = 0$ and $\theta = \pi/2$ is quite close to that of the potential flow. However, the pressure over the rear departs substantially from the potential flow. In practice the main flow leaves the surface at points like S_L or S_T and, from that point on, the pressure is much lower than the potential flow pressure. This means that the actual drag is far from zero. We will describe and explain these features in more detail on later pages.