Kelvin Ovals

Steady, planar potential flows around finite bodies can also be generated using point vortices. The classic example of such flows are the so-called **Kelvin ovals** generated by placing two free vortices of equal strength but opposite sign at two symmetric locations as shown in Figure 1. It follows that the combined

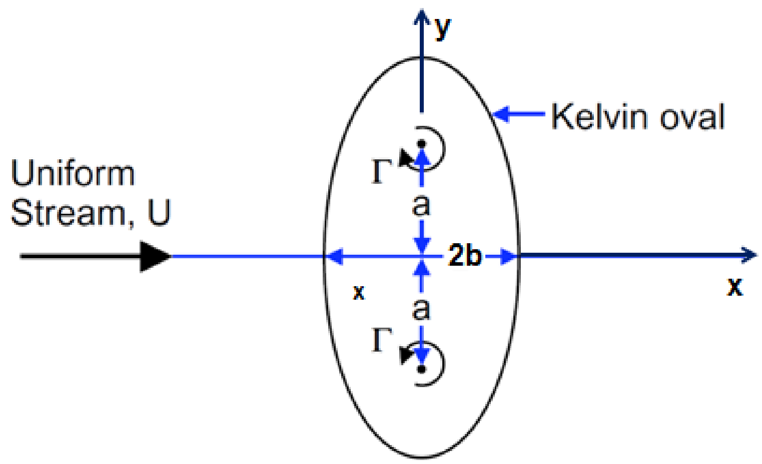

Figure 1: Two free vortices in a uniform stream generating a Kelvin oval.

velocity potential of the uniform stream and the two vortices will be given by

$$\phi \;=\; Ux - \frac{\Gamma}{2\pi}\theta_1 + \frac{\Gamma}{2\pi}\theta_2 \;=\; Ux - \frac{\Gamma}{2\pi}\arctan\frac{(y-a)}{x} + \frac{\Gamma}{2\pi}\arctan\frac{(y+a)}{x} \quad \text{(Bgdk1)}$$

and the velocities are

$$u \;=\; U + \frac{\Gamma(y-a)}{2\pi[x^2+(y-a)^2]} - \frac{\Gamma(y+a)}{2\pi[x^2+(y+a)^2]} \quad \text{(Bgdk2)}$$

$$v \;=\; -\frac{\Gamma x}{2\pi[x^2+(y-a)^2]} + \frac{\Gamma x}{2\pi[x^2+(y+a)^2]} \quad \text{(Bgdk3)}$$

Then, the half-thickness, b, of the oval on the x axis may be evaluated by locating the stagnation point which occurs when

$$u_{y=0} \;=\; 0 \quad \text{or where} \quad x \;=\; \pm\, b \quad \text{where} \quad b \;=\; \left[\frac{\Gamma a}{\pi U} - a^2\right]^{\frac{1}{2}} \quad \text{(Bgdk4)}$$

Clearly, the Kelvin ovals become increasingly thin in the x direction as the parameter $\Gamma/\pi aU$ is decreased. Indeed the thickness, b becomes zero when $\Gamma/\pi aU$ decreases to unity. Typical shapes for Kelvin ovals are shown in Figure 2. For values of $\Gamma/\pi aU > 1$ the shape is an oval like that of Figure 2(a) with increasing thickness for increasing $\Gamma/\pi aU$. For $\Gamma/\pi aU = 1$ it has a double teardrop shape like that of Figure 2(b). For $\Gamma/\pi aU < 1$ the oval decomposes into two symmetric cylindrical shapes as shown in Figure 2(c).

As a postscript, note that because of the symmetry the x axis in all of these flows is a streamline. Therefore we could replace the lower half-plane by a solid object and thus obtain the flow caused by a vortex in the proximity of a horizontal wall. In effect, one can interpret the vortex at $y = -a$ as an image vortex used

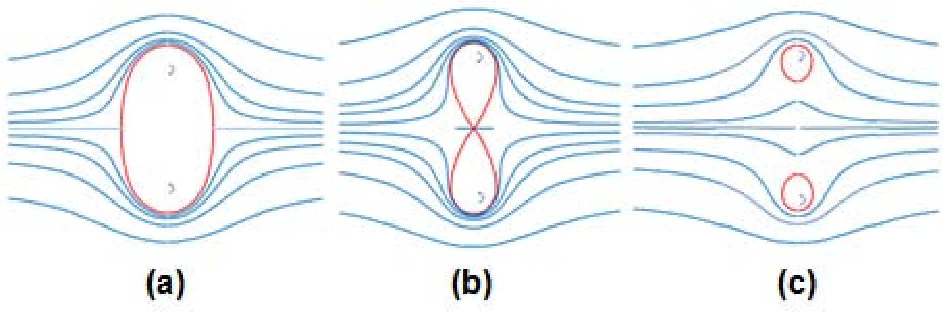

Figure 2: Three Kelvin ovals (in red) for various values of $\Gamma/\pi a U$.

to generate a plane of symmetry and therefore a solid boundary on the x axis. This is one of many such potential flows near boundaries that can be generated by the method of singularities supplemented by the use of images. We address this general methodology in a following section.