Sources and Sinks

Another of the most basic potential flows is that due to a source (or sink). The source or sink that produces a planar flow is called a **line source (or sink)** since it must extend indefinitely in a line perpendicular to the plane of the flow. This corresponds to the general solution (equations (Bgdb10 - 13)) in which $C_{40}C_{10} = C$ is non-zero while all the other C coefficients are zero. This leaves

$$\phi = C \ln r \quad ; \quad u_r = \frac{C}{r} \quad ; \quad u_\theta = 0 \quad ; \quad \psi = C\theta \tag{Bgdd1}$$

The volume flow rate, Q, emanating from this line source per unit depth normal to the plane of the flow is then

$$Q = \frac{C}{r} 2\pi r = 2\pi C \tag{Bgdd2}$$

and we therefore replace the constant, C, with Q so that the characteristics of a line source at the origin are

$$\phi = \frac{Q}{2\pi} \ln r \quad ; \quad u_r = \frac{Q}{2\pi r} \quad ; \quad u_\theta = 0 \quad ; \quad \psi = \frac{Q\theta}{2\pi} \tag{Bgdd3}$$

and a **line sink** simply has a negative rather than positive value of Q.

We will have occasion to place sources or sinks at other locations within a Cartesian coordinate system. Then, for example, a line source placed at x_0, y_0 will induce a velocity potential given by

$$\phi = \frac{Q}{4\pi} \ln\left\{(x - x_0)^2 + (y - y_0)^2\right\} \quad ; \quad \psi = \frac{Q}{2\pi} \arctan\left\{\frac{(y - y_0)}{(x - x_0)}\right\} \tag{Bgdd4}$$

Though most of the present text will be concerned with planar potential flows, we should note that a general, three-dimensional source, a so-called **point source** will clearly have a potential given by

$$\phi = \frac{Q}{4\pi r} \quad ; \quad u_r = \frac{Q}{4\pi r^2} \tag{Bgdd5}$$

where Q is now the total volume flow rate from the source. Note that ϕ now dies off more rapidly with r than for a line source (like $1/r$ rather than $\ln r$) and similarly u_r dies off more rapidly (like $1/r^2$ rather than $1/r$). We will return to three-dimensional flows in a later section.

As a footnote we might mention that a good first example of the use of superposition is to combine a free vortex with a line sink to generate the basic planar flow associated with a hurricane. As depicted by the plan view in Figure 1, we generate the flow of a hurricane outside of the core by combining a free vortex of strength, Γ, with a line sink to yield a flow given by

$$\phi = \frac{Q}{2\pi} \ln r + \frac{\Gamma}{2\pi}\theta \quad ; \quad u_r = \frac{Q}{2\pi r} \quad ; \quad u_\theta = \frac{\Gamma}{2\pi r} \quad ; \quad \psi = \frac{Q\theta}{2\pi} - \frac{\Gamma}{2\pi} \ln r \tag{Bgdd6}$$

where the strength, Q, is negative. In a hurricane the flow at ground level experiences a sink because of the updraft in the central core. The above solution only applies to the potential flow outside of the core; the core is dominated by viscous effects which help cause solid-body rotation in that inner region much as was described for a real vortex. We will assume that the radius of the core, $R = 40\ m$, that the sink strength is $Q = -5000\ m^2$, that the pressure difference between the pressure far away and the pressure at the edge of the core is $1500\ kg/m^2s$ and that the air density everywhere is about $1.2\ kg/m^3$

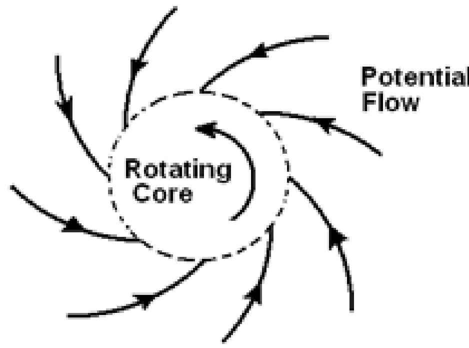

Figure 1: Plan view of a hurricane.

Since the flow outside the core is potential (or at least modeled as such) Bernoulli's equation applies so that, neglecting gravity effects, the pressure at very large radii, p_∞ and that at the boundary of the core, p_c are related by

$$p_\infty \;=\; p_c + \frac{\rho}{2}\left\{u_r^2 + u_\theta^2\right\}_{r=R} \qquad \text{(Bgdd7)}$$

and since $u_r = -5000/(2\pi R) = 19.9\ m/s$ and the pressure difference, $p_\infty - p_c = 1500\ kg/m^2 s$ it follows that

$$u_\theta \;=\; \left[\frac{3000}{1.2} - 19.9^2\right]^{\frac{1}{2}} \;=\; 45.9\ m/s \qquad \text{(Bgdd8)}$$

Hence we see that the rotation of the hurricane is associated with the low pressure in the center and the sink flow that accompanies it. The highest winds (in this case of 50 m/s) occur at the edge of the core, primarily as a result of that rotation of 1.15 $radians/s$.