Incompressible Flows with External Energy Exchange

In discussing the application of the first law of thermodynamics to the steady flow of an incompressible, inviscid fluid, we recognized that the result was separation of the heat transfer processes and the fluid flow. The result was that

$$\Delta w \;=\; \Delta \left\{ \frac{p}{\rho} + \frac{|\underline{u}|^2}{2} + gy \right\} \;=\; g\Delta H \tag{Bff1}$$

where Δw is the external work done on a Lagrangian volume per unit mass (excluding the work done by surface forces and by gravity). We now explore the implications of this result for incompressible internal flows in which mechanical energy or work is exchanged between the fluid and mechanical moving surfaces. When a Lagrangian fluid mass passes through a device like a pump or turbine with blades which do work on the fluid or extract work from the fluid, then that work changes the total head of the fluid mass as it passes through the device. Specifically, if the rate of work done *on the fluid* per unit time by the pump or other active device is denoted by $\dot{W}$ and the mass flow rate of fluid through the device is denoted by m, then the work done on the fluid per unit mass is $\dot{W}/m$ and it follows that the increase in total head, H, across the pump, turbine or other device is equal to $\dot{W}/(gm)$:

$$\Delta H \;=\; H_2 - H_1 \;=\; \left\{ \frac{p}{(\rho g)} + \frac{|\underline{u}|^2}{(2g)} + y \right\}_2 \;-\; \left\{ \frac{p}{(\rho g)} + \frac{|\underline{u}|^2}{(2g)} + y \right\}_1 \;=\; \frac{\dot{W}}{(gm)} \tag{Bff2}$$

where the subscripts 1 and 2 respectively refer to the inflow to and discharge from the pump or other device. Clearly a device such as a turbine which removes energy from the fluid has a negative value of $\dot{W}$. This relation is also commonly written as

$$\dot{W} \;=\; Q \times \rho g \times \Delta H \tag{Bff3}$$

where Q is the volume flow rate.

As in the case of passive hydraulic components, we can also modify this result to accommodate the viscous losses (as well as other losses) within a device like a pump or turbine. First we address the case in which work is done on the fluid in a device like a pump in which energy is inserted through a rotating shaft (or other mechanical device). In the ideal inviscid case addressed above, all that work would be converted to head rise, ΔH, according to equation (Bff3). In a real device viscous and other irreversible frictional effects mean that only a fraction, η_P, of the work, $\dot{W}$, gets converted to head rise; the remainder of the work is irreversibly converted to heat. The fraction, η_P, is the **pump efficiency** given by

$$\eta_P = \frac{\rho g \Delta H}{\dot{W}} \tag{Bff4}$$

where in the case of a pump both $\Delta H = H_2 - H_1$ and $\dot{W}$ are positive quantities. Efficiencies of large pumps operating at their design condition can be as high as 90% though this drops off the further off design the pump is operated. Smaller pumps operate at lower Reynolds numbers and tend, therefore, to have lower efficiencies. However, even the crudest pumps have efficiencies greater than 60%. Much more detail is given elsewhere in these pages.

While the fluid mechanics of centrifugal and axial flow pumps are explored elsewhere in much more detail, it is worth noting that these qualitatively function in the following way. The shaft rotates a bladed component called an impeller that increases the velocity of the fluid and therefore the velocity component of the total head. After exiting the impeller, this dynamic head usually needs to be converted to pressure using a diffuser. Viscous losses occur during both steps though the diffuser losses normally dominate.

In the case of a turbine, the fluid is used to rotate the impeller (or runner) and the total head drop through the turbine, $(-\Delta H)$, produces work, $(-\dot{W})$, that is transmitted out through the shaft of the turbine. In the absence of viscous and other irreversible losses, equation (Bff3) is the relation between $-\Delta H$ and the ideal $-\dot{W}$ that could be produced. However, those losses mean that only a fraction, η_T, of that work can actually be generated. Thus, the turbine efficiency, η_T, is given by

$$\eta_T = \frac{(-\dot{W})}{\rho g(H_1 - H_2)} \tag{Bff5}$$

When operating at their design condition, large turbines can have efficiencies greater than 90%.