Axisymmetric Stokes Flow

The equations governing axisymmetric Stokes flows are as follows. As in the corresponding incompressible *potential flow* (see section (Bgfa)), the continuity equation can be satisfied by defining the *Stokes streamfunction*, ψ, given by

$$u_z \;=\; \frac{1}{r}\frac{\partial\psi}{\partial r} \qquad ; \qquad u_r \;=\; -\frac{1}{r}\frac{\partial\psi}{\partial z} \tag{Blg1}$$

where (r, z) are the cylindrical coordinates of the axisymmetric flow (which has no gradients in the circumferential or θ direction) and (u_r, u_z) are the fluid velocities in the (r, z) directions. This Stokes streamfunction should not be confused with the streamfunction used in planar incompressible flows. The equations of motion for incompressible axisymmetric Stokes flows follow by eliminating the inertial terms in equations (Bhg1) and (Bhg3):

$$-\frac{\partial p}{\partial r} + f_r + \mu\left[\nabla^2 u_r - \frac{u_r}{r^2}\right] \;=\; 0 \tag{Blg2}$$

$$-\frac{\partial p}{\partial z} + f_z + \mu\nabla^2 u_z \;=\; 0 \tag{Blg3}$$

where the operator, ∇^2, is

$$\nabla^2 \;\equiv\; \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{Blg4}$$

and the force field terms can be absorbed into the pressure terms provided the force field is conservative. Thus, the pressure can be eliminated from equations (Blg2) and (Blg3) to obtain the following governing equation for axisymmetric Stokes flows:

$$-\frac{\partial p}{\partial r} + \mu\left[\nabla^2 u_r - \frac{u_r}{r^2}\right] \;=\; 0 \tag{Blg5}$$

$$-\frac{\partial p}{\partial z} + \mu\nabla^2 u_z \;=\; 0 \tag{Blg6}$$

Solutions to axisymmetric Stokes flows are best constructed by the superposition of individual Stokes flow singularites (as described in section (Blc)) distributed along the axis of the flow.