Turbulent Couette Flow

Prandtl's mixing length model applied to turbulent Couette flow produces the following result with

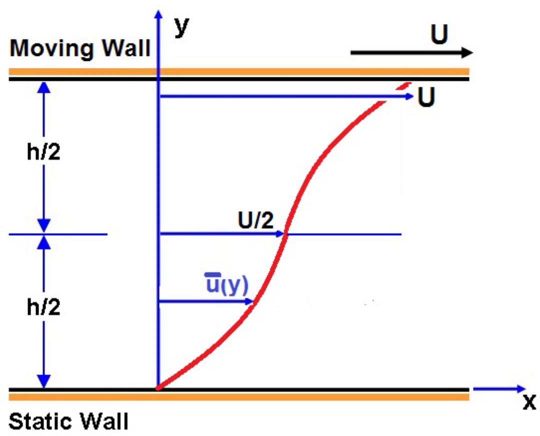

Figure 1: Notation for turbulent Couette flow.

notation as described in figure 1. We focus on the lower half of the flow since the mean velocity, $u(y)$, must be symmetric about $y = h/2$ with

$$\overline{u}(h-y) \quad = \quad U - \overline{u}(y) \quad \text{and} \quad \overline{u}(h/2) \quad = \quad U/2 \tag{Bki1}$$

and satisfy the no-slip conditions $\overline{u}(h) = U$ and $\overline{u}(0) = 0$. As in any Couette flow the shear stress, σ_{xy}, must be uniform across the flow and therefore from equation (Bkg7)

$$\frac{\partial(\overline{u'v'})}{\partial y} = \nu \frac{\partial^2 \overline{u}}{\partial y^2} \quad \text{or} \quad \overline{u'v'} = \nu \frac{d\overline{u}}{dy} + C \tag{Bki2}$$

where we have assumed there is no pressure gradient in the x direction, C is an integration constant and we have replaced the partial derivatives with d/dy since all quantities vary only with y and not with x. Then substituting the Prandtl mixing length model, namely

$$\overline{u'v'} \; \approx \; \kappa^2 y^2 \left(\frac{d\overline{u}}{dy}\right)^2 \tag{Bki3}$$

so the equation that must be solved to find $\overline{u}(y)$ becomes

$$\kappa^2 y^2 \left(\frac{d\overline{u}}{dy}\right)^2 = \nu \frac{d\overline{u}}{dy} + C \tag{Bki4}$$