Fluid Acceleration far from the Body

In the preceding sections, the introduction and the section discussing added mass, we have confined the presentation to those cases in which the fluid far from the body was at rest. Clearly, unless this is the case the total kinetic energy integral, T, defined by equation (Bmba1) becomes infinite and the subsequent analysis becomes meaningless. We turn attention now to those circumstances in which the fluid far from the body is either (a) moving with a constant, uniform velocity or (b) accelerating.

Examine first case (a) and consider the case in which the fluid far from the body has some uniform and constant velocity, W_i. It is clear that since the inertial forces cannot be altered by a simple Galilean transformation that the appropriate definition of T under those circumstance should be

$$T = \frac{\rho}{2}\int_V (u_i - W_i)(u_i - W_i) dv \qquad \text{(Bmbc1)}$$

The value of this integral is then finite and the problem is resolved. In other words the appropriate velocity, u_i, to be used in equation (Bmba1) is the velocity of the fluid relative to the fluid velocity far from the body. This leads to no change in the formulation of the fluid inertial forces. Thus a more universal expression for those forces is

$$F_i = -M_{ij}\frac{d(U_j - W_j)}{dt} = -M_{ij}\frac{dU_j}{dt} \qquad \text{(Bmbc2)}$$

In other words, since the derivative of W_j is zero we recover the previous expression.

Case (b) in which W_j is a function of time is, however, more complicated yet important because it is a common occurrence and because some of the experiments carried out to measure the inertial forces use an accelerated fluid flow rather than an accelerated body. We begin with a case (a) with a constant, uniform velocity, W_j, $j = 1, 2, 3$ far from the body whose center of volume is moving at a velocity, U_j, $j = 1 \rightarrow 6$ and which is accelerating with components, A_j. Equation (Bmbc2) is appropriate and superposability is required. Then we choose to apply an additional, globally uniform acceleration, dW_j/dt, $j = 1, 2, 3$ to both the body and the fluid so that the new acceleration of the body is $A_j + (dW_j/dt)$. It transpires as long as we begin with a superposable set of equations governing the fluid flow (for example, the equations of potential flow or Stokes' flow) and a set of boundary conditions that are also superposable, then the equations governing this new flow with the added global acceleration are identical to those without the global acceleration except that where the pressure, p, occurs it is replaced by $p - \rho x_j(dW_j/dt)$. Consequently the forces that the fluid exerts on the body are identical except for an additional contribution in the flow with the additional global acceleration due to the additional pressure $-\rho x_j(dW_j/dt)$. When this is integrated over the surface of the body the additional, buoyancy-like force on the body becomes $\rho V_b(dW_j/dt)$ where V_b is the volume of fluid displaced by the body. Therefore the inertial force becomes

$$F_i = -M_{ij}A_j + \rho V_b\frac{dW_i}{dt} \qquad \text{(Bmbc3)}$$

where the acceleration of the body, dU_j/dt, is now

$$\frac{dU_j}{dt} = A_j + \frac{dW_j}{dt} \qquad \text{(Bmbc4)}$$

and dW_j/dt is the acceleration of the fluid far from the body. Substituting for A_j in equation (Bmbc3) produces the final result for the flow with fluid acceleration far from the body namely

$$F_i = -M_{ij}\frac{dU_j}{dt} + (M_{ij} + \rho V_b\delta_{ij})\frac{dW_i}{dt} \quad \text{for} \quad j = 1, 2, 3 \qquad \text{(Bmbc5)}$$

where δ_{ij} is the Kroneker delta ($\delta_{ij} = 1$ for $i = j$, $\delta_{ij} = 0$ for $i \neq j$. Therefore the "added mass" associated with the **fluid acceleration**, dW_j/dt, is the sum of the true added mass, M_{ij}, and the mass of the displaced fluid, ρV_b. This distinction is essential in interpreting and comparing the results of experiments designed to measure inertial effects. For example, in the experiments of Keulegan and Carpenter (1958) the body is held at rest while the fluid far from the body undergoes sinusoidal acceleration so that the fluid inertial effects are proportional to $(M_{ij} + \rho V_b)$. In contrast, in the experiments of Skop, Ramberg and Ferer (1976) the body is accelerated while the fluid far away is not and therefore their fluid inertial effects are proportional to M_{ij}. The term "added mass" should be reserved for M_{ij} in order to avoid confusion.