Introduction to Added Mass

Whenever acceleration is imposed on a fluid flow either by acceleration of a body in the fluid or by acceleration externally imposed on the fluid, additional fluid forces will act on the surfaces in contact with the fluid. These fluid inertial forces can be of considerable importance in many practical situations. In this and the sections which follow we will review the state of knowledge of these forces and, in particular, identify the *added mass matrices* that can be used to characterize them.

Perhaps the most fundamental view of the phenomenon of *added mass* is that it defines the necessary work that is needed to change the kinetic energy associated with a fluid motion. Any fluid motion such as that which occurs when a body moves through the fluid implies a certain, positive, non-zero kinetic energy associated with the fluid motion. We will confine attention to an incompressible fluid of density, ρ, in which case the total kinetic energy, T, is given by

$$T \;=\; \frac{\rho}{2}\int_V (u_1^2+u_2^2+u_3^2)dv \;=\; \frac{\rho}{2}\int_V u_j u_j dv \tag{Bmba1}$$

where u_j, $j=1,2,3$ are the Cartesian components of the fluid velocity and V is entire volume or domain of fluid. If the motion of the body is one of steady rectilinear translation at a velocity U through a fluid otherwise at rest then clearly the total kinetic energy is finite and constant; it must in fact be equal to the work that had to be done on the body to get it up to that velocity after starting form rest with all velocities equal to zero. Moreover it is likely that the fluid velocities, u_j, will, in some manner, be proportional to U in which case T will be proportional to U^2. In such cases

$$T \;=\; \frac{\rho U^2}{2} I \quad \text{where} \quad I \;=\; \int_V \frac{u_j}{U}\frac{u_j}{U} dv \tag{Bmba2}$$

and the integral I will be some simple, unvarying number. This is indeed the case with some fluid flows such as potential flow or Stokes' flow at low Reynolds numbers. However, it may not be true for the complicated, vortex-shedding flows that occur at intermediate Reynolds numbers.

Now consider that the body begins to accelerate or decelerate. The kinetic energy in the fluid will also begin to change as U changes. If the body accelerates the kinetic energy will increase in all probability. However, this energy must be supplied through work done by the body on the fluid in order to increase T. Moreover the rate of additional work required is simply the rate of change of T, dT/dt. This additional work is experienced by the body as an additional drag, such that the rate of additional work done, $-FU = dT/dt$, where the negative sign results from the choice that F is positive in the same direction as U. Moreover, if the pattern of the flow is constant so that I remains unchanged

$$F \;=\; -\frac{1}{U}\frac{dT}{dt} \;=\; -\rho I \frac{dU}{dt} \;=\; -M\frac{dU}{dt} \tag{Bmba3}$$

This force has the same form and sign as that required to accelerate the solid mass, m, of the body, namely $m\ dU/dt$. Consequently it is often convenient to consider the mass, $M = \rho I$, as an "added mass" that is being accelerated along with the body. Of course, there is no such identifiable fluid mass; rather all of the fluid is accelerating to some degree as the total kinetic energy is increasing.

Note, parenthetically and obviously, that F is not the only drag force experienced by the body. During steady translation through a real viscous fluid there is, of course, a steady drag associated with the necessary work which must be done to balance the steady rate of dissipation of energy in the viscous fluid. When the body accelerates there will be a similar though not necessarily equal drag associated with the instantaneous magnitude of U. Furthermore there may be delayed effects associated with the entire previous history of translation as exemplified by the Basset force.

Perhaps the simplest touchstone evaluations of the added mass, M, are those for the planar potential flow past a cylinder of radius R (equation (Bgdh1)) for which the added mass per unit length, M^*, is

$$M^* \;=\; \rho \int_R^\infty \int_0^{2\pi} \left[\left\{ \frac{1}{U} \frac{\partial \phi}{\partial r} \right\}^2 + \left\{ \frac{1}{Ur} \frac{\partial \phi}{\partial \theta} \right\}^2 \right] r d\theta dr \;=\; \pi \rho R^2 \qquad \text{(Bmba4)}$$

and the axisymmetric potential flow past a sphere of radius R (equation (Bgdn9)) for which

$$M \;=\; \rho \int_R^\infty \int_0^{\pi} \left[\left\{ \frac{1}{U} \frac{\partial \phi}{\partial r} \right\}^2 + \left\{ \frac{1}{Ur} \frac{\partial \phi}{\partial \theta} \right\}^2 \right] 2\pi r^2 \sin\theta d\theta dr \;=\; \frac{2\pi}{3} \rho R^3 \qquad \text{(Bmba5)}$$

Thus the added mass of a cylinder in potential flow is equal to the mass of fluid displaced by the body whereas the added mass of a sphere in potential flow is equal to one half of the mass displaced by the sphere. Indeed there is, in general, no relation between the added mass and the displaced mass. For example an infinitely thin flat plate accelerated normal to itself has zero displaced mass but its added mass in potential flow is equal to the mass of a circular cylinder of fluid with a diameter equal to the width of the plate. This example also demonstrates that the added mass depends not only on the geometry of the body but also on the direction of the acceleration for the added mass of the flat plate in the direction parallel with its plane is zero in potential flow. To broaden the picture further it is clear that, for a general body, acceleration in one direction may lead not only to a force in that direction but also to forces in other directions. Consequently, in general, we must define a relationship between the accelerations and the resulting forces. This is pursued in the section which follows.