Additional effects on the Added Mass

In this section a few additional effects on the added mass are briefly described:

Effect of a nearby solid boundary: The presence of a solid boundary close to the body being accelerated usually increases the added mass of that body because it generates regions of larger fluid accelerations than in the absence of that boundary. The increase is readily observed in some of the data listed in the section (Bmbd). Sometimes the increase is very large as in the following example.

Consider the planar flow example of a flat plate of width, $2a$, near the ocean bottom as depicted in Figure 1. A vertical force, F, is applied at the midpoint of the plate to lift it away from the ocean floor so that

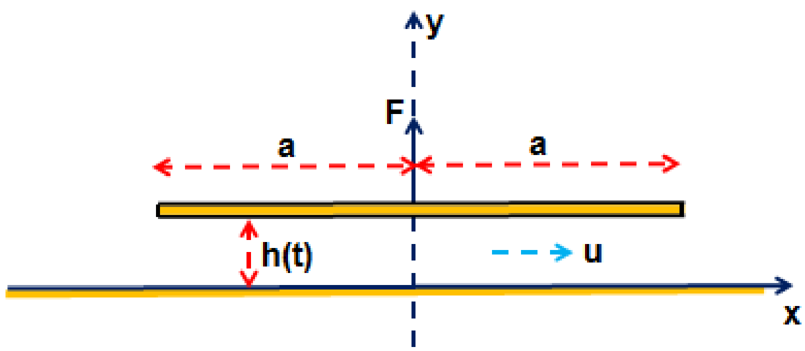

Figure 1: Flat plate near the ocean bottom.

the gap between the plate and the ocean floor, $h(t)$, is to increase with time, t. The upward velocity and acceleration of the plate are therefore dh/dt and d^2h/dt^2. We assume that h is small compared with a so that the velocity of the plate, dh/dt, will generate much larger fluid velocities in the gap, $u(t)$, and those velocities will be predominantly in the x direction. Then, if the fluid is incompressible and we assume a uniform velocity profile in the gap, conservation of mass plus the symmetry about $x = 0$ requires that

$$hu = -x\frac{dh}{dt} \qquad \text{(Bmbf1)}$$

Moreover, the momentum equation for the fluid in the gap in the absence of frictional, viscous effects yields

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + 2u\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial (hu)}{\partial t} = 0 \qquad \text{(Bmbf2)}$$

where $p(x,t)$ is the pressure in the gap. Substitution for u and integration yield the following form for the pressure distribution in the gap:

$$p = p_e + \frac{\rho}{2}(a^2 - x^2)h\frac{\partial^2}{\partial t^2}\left\{\frac{1}{h}\right\} \qquad \text{(Bmbf3)}$$

where p_e is the pressure at the edges, $x = \pm a$. While the gap is small p_e will be approximately equal to the ambient pressure far from the plate, p_a, and the pressure on the upper side of the plate will also be approximately p_a. Consequently by integrating the pressure difference, $p - p_a$, we can evaluate the force that the fluid applies to the plate and therefore the equal and opposite force, F, that has to be applied to the plate to lift it upward:

$$F = \frac{2\rho a^3}{3h}\left\{\frac{\partial^2 h}{\partial t^2} - \frac{2}{h}\left(\frac{\partial h}{\partial t}\right)^2\right\} \qquad \text{(Bmbf4)}$$

This is the magnitude of the inertial or added mass force acting downward on the plate that has to be overcome in order to move the plate upward. It should be compared with the similar force in the absence of the ocean bottom, namely $\rho\pi a^2\partial^2 h/\partial t^2$ where $\partial^2 h/\partial t^2$ is again used to represent the vertically upward acceleration. Clearly the magnitude of the former given by equation (Bmbf4) is much larger than the latter by the large factor, a/h. Of course the preceding analysis ceases to be valid when h approaches a. But it is clear from equation (Bmbf4) that as the plate is raised the added mass (per unit breadth normal to Figure 1) begins at a very large value of the order of $\rho a^3/h$ when h is small and will rapidly decrease as h increases eventually approaching a value of the order of ρa^2 when h approaches a.

This analysis is pertinent when any flat object is lying on a bed (the ocean floor). Even if only portions of the object are in contact with the bed, one can surmise that there is some average initial separation, h_0, between the object and the bed (silting on a river bed or the ocean floor could of course make h_0 even smaller). Then if an upward lifting force is applied at, say, $t = 0$, the initial force necessary to cause any upward motion may be very large indeed. For example, suppose the upward force, F, that is applied is constant and we neglect the mass of the plate itself. Then integration of the equation of motion (Pbf4) yields a time history given by

$$h = \frac{h_0}{\cos\left\{\left(\frac{3F}{2\rho a^3}\right)^{\frac{1}{2}} t\right\}} \qquad \text{(Bmbf5)}$$

where initial conditions $h = h_0$ and $dh/dt = 0$ at $t = 0$ have been used. Clearly this leads to a catastrophic release from the bottom in which the upward acceleration increases with time. In practice it is unlikely that a constant lifting force could be maintained under these circumstances. Nevertheless such a sudden motion is a familiar phenomenon. In deep sea recovery it can cause considerable strategic difficulties; for example, in the deep sea recovery of airplanes that sudden motion can cause the airplane to come apart.

Effect of a nearby free surface: Unlike the presence of a solid boundary, the presence of a nearby free surface adds considerably to the complexity of the problem primarily because the boundary condition is non-linear and therefore superposability is not satisfied. As a consequence the dynamic behavior of bodies on or near a free surface is a specialized area and here we will confine the discussion to a few brief remarks. The principal difficulty is that any prior translational or rotational motion generates surface waves which complicate the fluid flow. Only if the body velocity, U, is sufficiently slow so that the Froude number is much smaller than unity are the waves sufficiently small to be neglected. In the case of floating bodies the reader is referred to the reviews by Wehausen (1971), Newman (1970) and Ogilvie (1964). Submerged bodies are only slightly easier to handle and some data for the circumstance when the body is accelerating from rest is given in the preceding sections.