Measurement of Transfer Matrices

Any experiment designed to measure the transfer matrix of a pump or other hydraulic component needs to accumulate at least two (and often many more) linearly independent "sets" of data for each frequency and mean operating state. We will denote the number of sets by I and the sets therefore generate a data block comprising complex values of the fluctuating total pressures and mass flow rates represented by

$$\tilde{p}_1^{T\ i} \quad , \quad \tilde{p}_2^{T\ i} \quad , \quad \tilde{m}_1^i \quad , \quad \tilde{m}_2^i \quad \text{for} \quad i \ = \ 1,\ 2\ ,3\ ...I \tag{Bngl2}$$

Then, to obtain the matrix $[T]$

$$\left\{ \begin{matrix} \tilde{p}_2^T \\ \tilde{m}_2 \end{matrix} \right\} = \begin{bmatrix} T_{11}\, T_{12} \\ T_{21}\, T_{22} \end{bmatrix} \left\{ \begin{matrix} \tilde{p}_1^T \\ \tilde{m}_1 \end{matrix} \right\} \tag{Bngl1}$$

that best fits that block of data, :

$$T_{11} \ = \ S_8(S_5\ S_2 - S_3\ S_6) \quad \text{and} \quad T_{12} \ = \ S_8(S_6\ S_1 - S_2\ \overline{S}_3) \tag{Bngl3}$$

and

$$T_{21} \ = \ S_8(S_5\ S_4 - S_3\ S_7) \quad \text{and} \quad T_{22} \ = \ S_8(S_7\ S_1 - S_4\ \overline{S}_3) \tag{Bngl4}$$

where

$$S_1 \ = \ \sum_{i=1}^{I} \tilde{p}_1^{T\ i}\ \overline{\tilde{p}_1^{T\ i}} \quad , \quad S_2 \ = \ \sum_{i=1}^{I} \tilde{p}_2^{T\ i}\ \overline{\tilde{p}_1^{T\ i}} \quad , \quad S_3 \ = \ \sum_{i=1}^{I} \tilde{m}_1^i\ \overline{\tilde{p}_1^{T\ i}}$$

$$S_4 \ = \ \sum_{i=1}^{I} \overline{\tilde{p}_1^{T\ i}}\ \tilde{m}_2^i \quad , \quad S_5 \ = \ \sum_{i=1}^{I} \tilde{m}_1^i\ \overline{\tilde{m}_1^i} \quad , \quad S_6 \ = \ \sum_{i=1}^{I} \tilde{p}_2^{T\ i}\ \overline{\tilde{m}_1^i}$$

$$S_7 \ = \ \sum_{i=1}^{I} \tilde{m}_2^i\ \overline{\tilde{m}_1^i} \quad , \quad S_8 \ = \ \frac{1}{S_5\ S_1 - S_3\ \overline{S}_3} \tag{Bngl5}$$

This procedure minimizes the sum of the squares of the spectral radii of the residues in the two equations (Bngl1) comprising the transfer function. While the minimum acceptable value for I is 2, experience has shown that, given the data noise and uncertainty in most hydraulic system measurements, values like 8 or sixteen are advisable. This requires at least two sources of excitation at each frequency and the ability to change the relative amplitudes and the phase between those two sources. Testing for independence between the modes of excitation is also needed.