System Analysis

As in the case of steady flows any one-dimensional analysis of the internal flows in a hydraulic system begins with the assumption that the flow at every cross-section of the incompressible, internal flow can be characterized by a single pressure, p (or total pressure, p^T), and a single mass flow rate, m, (or volume-averaged velocity, u). Then the system is subdivided into its component parts, each identified by its index, k, as shown in figure 1 where each component is represented by a box. The connecting lines do not depict

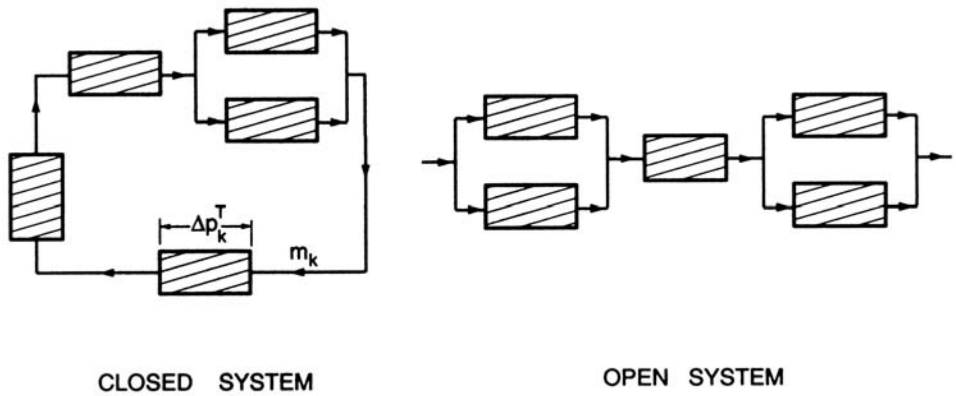

Figure 1: Hydraulic systems broken into components.

lengths of pipe which are themselves components. Rather the lines simply show how the components are connected. More specifically they represent specific locations at which the system has been divided up; these points will be called the nodes of the system and are denoted by the index, i. Typical and common components are pipeline sections, valves, pumps, turbines, boilers, and condensers. They can be connected in series and/or in parallel. Systems can be either open loop or closed loop as shown in figure 1. The mass flow rate through a component will be denoted by m_k and the change in the total pressure (or, equivalently, the total head) across the component will be denoted by Δp_k^T (or ΔH_k) defined as the total pressure (or head) at inlet minus that at discharge.

Next, the performance characteristic of the components considered in isolation needs to be identified. The performance characteristic is the relation between the total pressure drop (or total head drop) and the mass flow rate, namely the function $\Delta p_k^T(m_k)$ (or $\Delta H_k(m_k)$) as depicted graphically in figure 2. Some of these performance characteristics are readily anticipated. For example, a typical steady, incompressible flow through a horizontal pipe or passive fitting has a characteristic that is approximately quadratic (at least at high Reynolds number) with $\Delta p_k^T \propto m_k^2$. By definition of the loss coefficient, K_k,

$$\Delta p_k^T = \left\{ \frac{K_k}{2\rho^2 A_k^2} \right\} m_k^2 \quad \text{or} \quad \Delta H_k = \left\{ \frac{K_k}{2g\rho^3 A_k^2} \right\} m_k^2 \tag{Bnb1}$$

Other components such as pumps, compressors or fans may have quite non-monotonic characteristics. The slope of the characteristic, R_k^*, where

$$R_k^* = \frac{d\Delta H_k}{dm_k} = \frac{1}{\rho g} \frac{d\Delta p_k^T}{dm_k} \tag{Bnb2}$$

is known as the component resistance. However, unlike many electrical components, the resistance of most hydraulic components is almost never constant but varies with the flow, m_k.

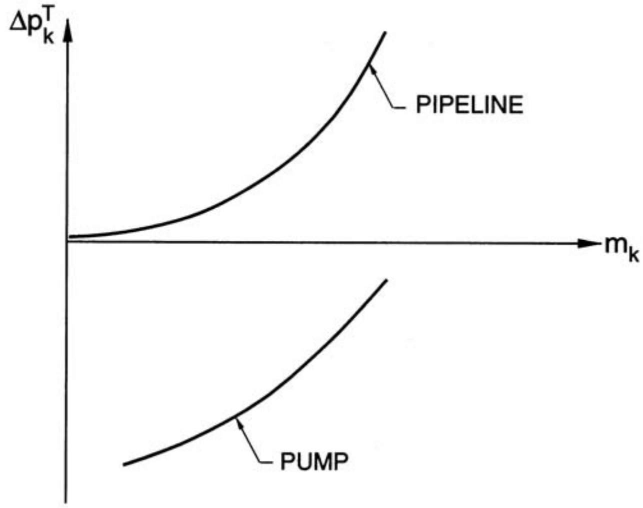

Figure 2: Typical component characteristics, $\Delta p_k^T(m_k)$.

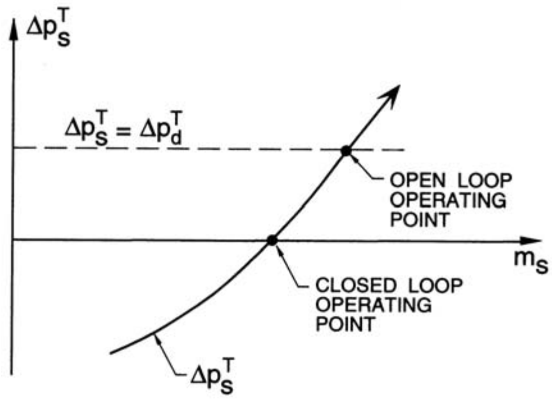

Figure 3: Typical system characteristic, $\Delta p_s^T(m_s)$, and operating point.

Components can then be combined to obtain the characteristic of groups of neighboring components or of the complete system. A parallel combination of two components simply requires one to add the flow rates at the same Δp^T (or ΔH), while a series combination simply requires that one add the Δp^T values of the two components at the same flow rate. In this way one can synthesize the total pressure drop, $\Delta p_s^T(m_s)$, for the whole system as a function of the system flow rate, m_s (into and out of the system). Such a system characteristic is depicted in figure 3. Depending on the analysis that is intended a subsequent step will be to apply whatever known boundary conditions are appropriate for the whole system.

In many contexts, the system equilibrium is depicted in a slightly different but completely equivalent way by dividing the system into two series elements, one of which is the *pumping* component, $k = pump$, and the other is the *pipeline* component, $k = line$. Then the operating point is given by the intersection of the *pipeline* characteristic, Δp_{line}^T, and the *pump* characteristic, $-\Delta p_{pump}^T$, as shown graphically in figure 4. Note that since the total pressure increases across a pump, the values of $-\Delta p_{pump}^T$ are normally positive. In most single phase systems, this depiction has the advantage that one can usually construct a series of quadratic *pipeline* characteristics depending on the valve settings. These *pipeline* characteristics are usually simple quadratics. On the other hand the pump or compressor characteristic can be quite complex.

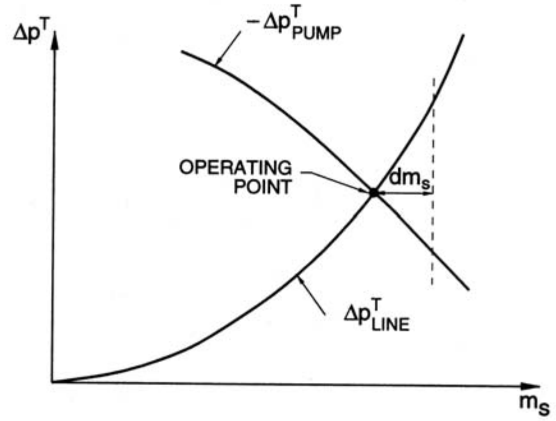

Figure 4: Alternate presentation of system equilibrium.