Manometer Oscillations

Uniform Area Manometer

As depicted in Figure 1, a manometer of uniform interior cross-sectional area, A, is filled with a length of incompressible, inviscid liquid equal to ℓ. The interior levels are then disturbed and released so that they oscillate freely and we wish to calculate the frequency of those **manometer oscillations**. Consider the

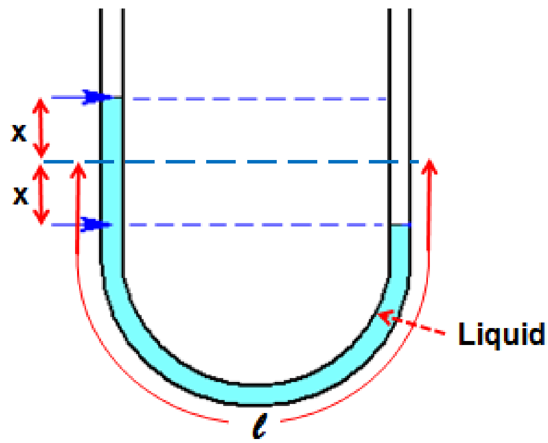

Figure 1: Simple manometer with uniform cross-sectional area.

moment when the level on the left-hand side is elevated by a distance x. Then the elevation at the right-hand side will be $-x$ and the velocity and acceleration of the fluid (considered positive in the clockwise direction) will be, respectively, dx/dt and d^2x/dt^2 throughout the length, ℓ. It follows that the total pressure, p^T, on the liquid surface on the left will be $p_a + \rho gx + \rho(dx/dt)^2$ and that on the liquid surface on the right will be $p_a - \rho gx + \rho(dx/dt)^2$ where p_a is the atmospheric pressure acting on both liquid surfaces. The difference in the total pressures (right minus left) is therefore $-2\rho gx$. If we neglect viscous, frictional effects, the unsteady Bernoulli equation states that this must be equal to the inertial term, $\rho\ell(d^2x/dt^2)$ and therefore:

$$-2gx \;=\; \ell\left\{\frac{d^2x}{dt^2}\right\} \qquad \text{(Bndb1)}$$

and this means that the motion of the levels is oscillatory with a natural frequency, ω (in radians/second) of

$$\omega \;=\; \left\{\frac{2g}{\ell}\right\}^{\frac{1}{2}} \qquad \text{(Bndb2)}$$

This frequency, which is essentially that of a simple pendulum, is commonly observed in everyday life. For example a pipe with a length of $\ell = 1m$ has a manometer frequency of about $4.4 rad/sec$ or about $0.7 Hz$. This allows it to be easily distinguished from other frequencies such as the acoustic frequencies of the pipe.

Manometer with different Areas

As a second example, consider a manometer with an interior cross-sectional area of A on one side and $2A$ on the other side is filled with incompressible, inviscid liquid as depicted in Figure 2. At rest, the side with

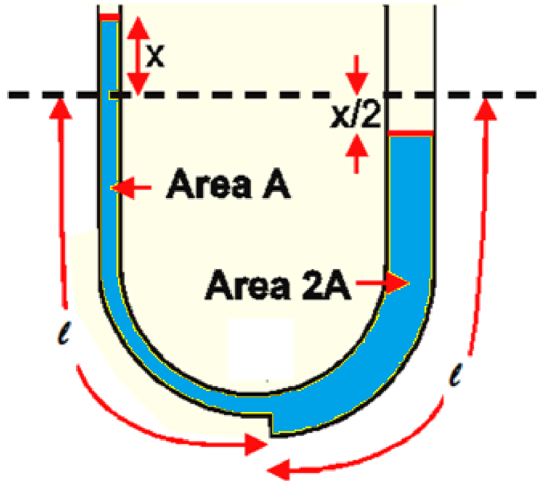

Figure 2: A manometer with two different cross-sectional areas.

area A and the side with area $2A$ both have a length equal to ℓ. The levels inside are then disturbed and released so that they oscillate and we wish to calculate the frequency of those **manometer oscillations**.

Neglecting any viscous, frictional effects, consider the moment when the fluid on the left hand side is elevated by a height x above the equilibrium position. Then, by conservation of mass, the elevation on the right-hand side will be $-x/2$ and the velocity and acceleration of the fluid (considered positive in the clockwise direction) will be, respectively, dx/dt and d^2x/dt^2 on the left and one half of that on the right. It follows that the total pressure, p^T, on the liquid surface on the left will be $p_a + \rho g x + \rho(dx/dt)^2$ and that on the liquid surface on the right will be $p_a - 0.5\rho g x + 0.25\rho(dx/dt)^2$ where p_a is the atmospheric pressure acting on both liquid surfaces. The difference in the total pressures (right minus left) is therefore

$$-\frac{3}{2}\rho g x - \frac{3}{4}\left\{\frac{dx}{dt}\right\}^2 \tag{Bndb3}$$

When the amplitude of the oscillatory motion is small the second term is a second-order effect and, if this is neglected along with any viscous, frictional effects, the unsteady Bernoulli equation states that the first term must be balanced by the sum of the inertial terms of the two sections so that

$$-\frac{3}{2}gx \;=\; \ell\left\{\frac{d^2x}{dt^2}\right\} + \ell\left\{\frac{1}{2}\frac{d^2x}{dt^2}\right\} \;=\; \frac{3}{2}\ell\left\{\frac{d^2x}{dt^2}\right\} \tag{Bndb4}$$

so that the frequency of this manometer becomes

$$\omega \;=\; \left\{\frac{g}{\ell}\right\}^{\frac{1}{2}} \tag{Bndb5}$$

Partially immersed straight pipe

As depicted in Figure 3, a straight pipe, open at both ends, is inserted in a large tank of liquid so that the submerged length is h. The level inside the pipe is displaced vertically and then released so that the level oscillates up and down; we wish to calculate the frequency of the oscillations. Consider the moment when the liquid surface in the tube is elevated by a height x above the equilibrium position. Then the velocity and acceleration of the fluid in the tube (considered positive upward) will be, respectively, dx/dt and d^2x/dt^2. It follows that the total pressure, p^T, on the liquid surface in the tube is $p_a + \rho g x + \rho(dx/dt)^2$

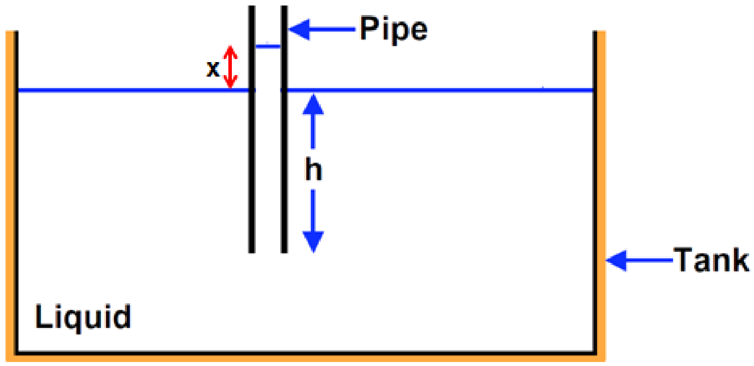

Figure 3: A straight pipe partially inserted in a large tank of liquid.

where p_a is the atmospheric pressure acting on all liquid surfaces. We will assume that the tank is very large so that both the elevation change in its surface and the velocity of its surface are negligible; consequently the total pressure of the tank surface is just p_a. Consequently the difference in the total pressures (tank surface minus interior tube surface) is $-\rho g x - \rho(dx/dt)^2$ and, as in the preceding example, the second term is a higher-order effect that will be neglected. Therefore the unsteady Bernoulli equation requires that the first term will be balanced by the inertial term. The contribution to the inertial term from the flow inside the tube will be $\rho h(d^2x/dt^2)$ and the acceleration in the tank is so small that the contribution from the tank will be negligible. The net result of the unsteady Bernoulli equation is therefore

$$-gx = h\left\{\frac{d^2x}{dt^2}\right\} \qquad \text{(Bndb6)}$$

so that the frequency of the oscillations in the tube is

$$\omega = \left\{\frac{g}{h}\right\}^{\frac{1}{2}} \qquad \text{(Bndb7)}$$