Bird Flight

Level bird flight is quite similar to fish swimming; the bird's wings perform motions very similar to those of the caudal fin or fluke of a fish or cetacean (see section (Dfd)) except that the bird must also produce a time-averaged lift to counteract its weight. A bird does that by modifying the motion that was depicted for fish locomotion in Figure 1 of section (Dfd) by adding a supplemental angle of inclination, α_0, as depicted in Figure 1. This supplemental angle of inclination, α_0, generates a time-averaged upward lift that keeps the bird aloft as we now demonstrate.

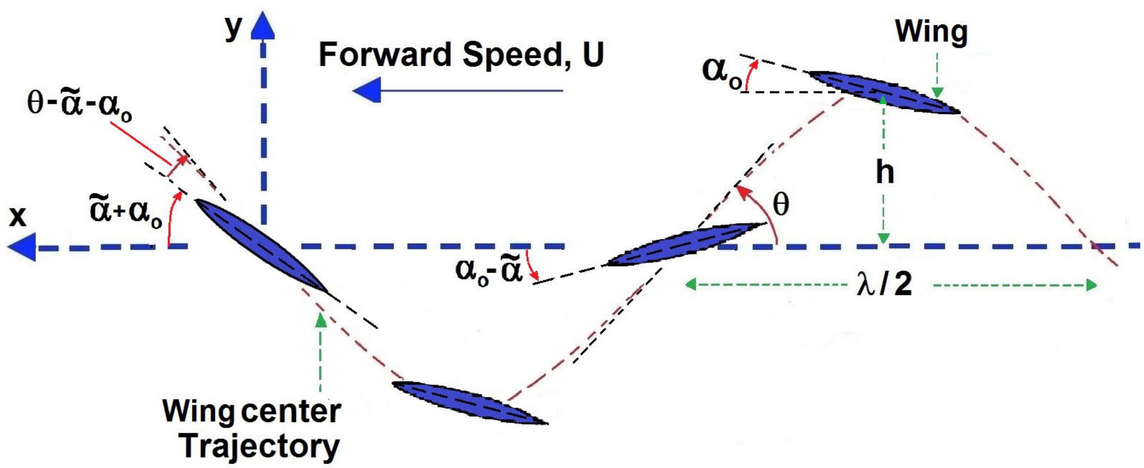

Figure 1: The motion of a foil simulating the dynamics of the wing of a flying bird.

It is convenient to develop a modified version of the analysis in section (Dfd) as follows. In this modified analysis the wing now performs a motion that is a combination of the forward translation at velocity, U, a heaving motion $y = h\sin\omega t$ and a pitching motion of the foil defined by a constant angle of incidence, α_0, plus the sinusoidal pitching motion, $\alpha = \tilde{\alpha}\cos\omega t$, so that the geometry of the motion is as depicted in Figure 1. Note again that the phase relationship of the heaving and pitching motions is such that the relative oscillating pitching motion, $\theta - \alpha$, lags the heaving motion by $\pi/2$. It follows that the inclination of the wing centerline trajectory with respect to the x axis (as shown in Figure 1) is θ where

$$\tan\theta = \frac{dy}{dx} = \frac{1}{U}\frac{dy}{dt} = \frac{\omega h}{U}\cos\omega t \qquad \text{(Dfe1)}$$

so that

$$\cos\theta = \left[1+\frac{\omega^2h^2}{U^2}\cos^2\omega t\right]^{-\frac{1}{2}} \quad \text{and} \quad \sin\theta = \frac{\omega h}{U}\cos\omega t\left[1+\frac{\omega^2h^2}{U^2}\cos^2\omega t\right]^{-\frac{1}{2}} \qquad \text{(Dfe2)}$$

The angle of incidence of the wing with respect to its *relative* motion through the air, α^*, is given by

$$\alpha^* = \theta - \alpha + \alpha_0 = \arctan\left\{\frac{\omega h}{U}\cos\omega t\right\} - \tilde{\alpha}\cos\omega t + \alpha_0 \qquad \text{(Dfe3)}$$

Moreover the magnitude of the velocity of relative motion, V, is given by

$$V^2 = U^2 + (\frac{dy}{dt})^2 = U^2 + \omega^2h^2\cos^2(\omega t) \qquad \text{(Dfe4)}$$

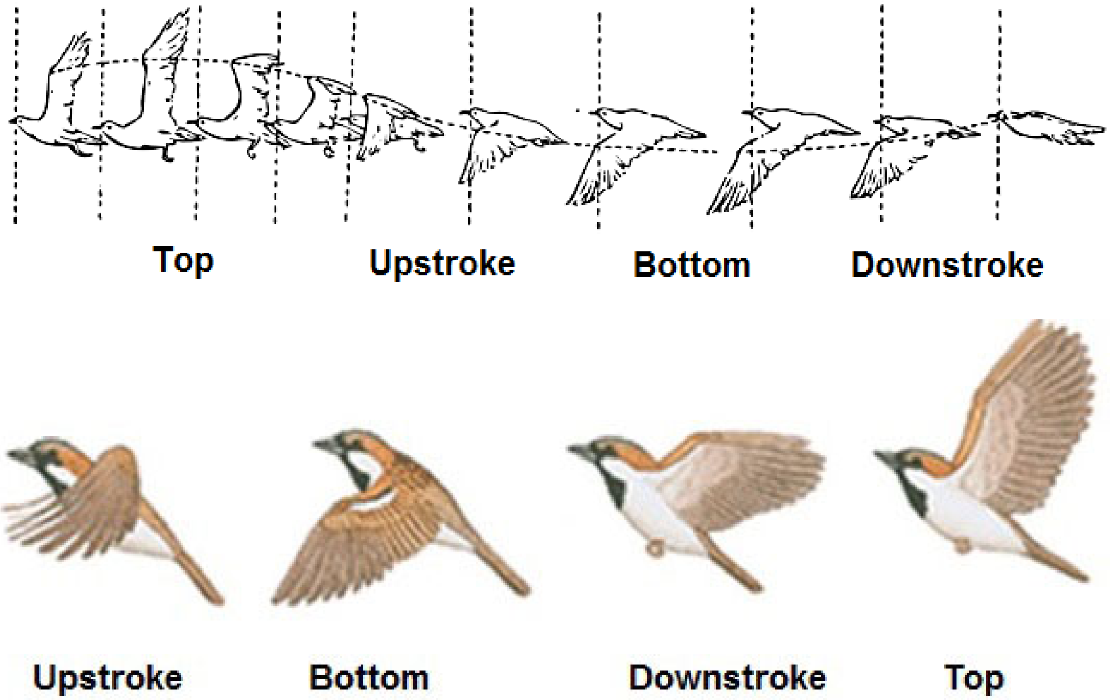

Figure 2: Examples of bird flight motion.

and therefore the instantaneous lift and drag forces acting on the wings (planform area, A_F), L and D, are given by $\frac{1}{2}\rho V^2 A_F C_L$ and $\frac{1}{2}\rho V^2 A_F C_D$ respectively where C_L and C_D are the lift and drag coefficients for the wings. It follows that the component of the instantaneous propulsive force in the x direction, F_x, is given by

$$\frac{F_x}{\frac{1}{2}\rho U^2 A_F} = \left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}\{C_L\sin\theta - C_D\cos\theta\} = \left\{C_L(\frac{\omega h}{U})\cos\omega t - C_D\right\}\left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}^{\frac{1}{2}} \tag{Dfe5}$$

The force in the y direction normal to the forward motion, F_y, is given by

$$\frac{F_y}{\frac{1}{2}\rho U^2 A_F} = \left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}\{C_L\cos\theta + C_D\sin\theta\} = \left\{C_L + C_D(\frac{\omega h}{U})\cos\omega t\right\}\left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}^{\frac{1}{2}} \tag{Dfe6}$$

Unlike the case for the neutrally-buoyant fish addressed in section (Dfd), the average of F_y over one cycle is non-zero and this average F_y must balance the weight of the bird in level flight.

We first examine the thrust, T, which is given by the mean value of F_x averaged over one cycle of the oscillation of the wings and therefore

$$T^* = \frac{T}{\frac{1}{2}\rho U^2 A_F} = \frac{1}{2\pi}\int_0^{2\pi}\left\{C_L(\frac{\omega h}{U})\cos\omega t - C_D\right\}\left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}^{\frac{1}{2}} d(\omega t) \tag{Dfe7}$$

Before going any further it is evident that the thrust produced by the motion of the wings is due to the component of the lift force in the x direction which may be positive during the entire cycle of wing motion. It is also clear that the drag on the wings will decrease that propulsive force and so we recognize again the importance of (1) a large lift/drag ratio (2) a well-designed wing cross-section and (3) as large an aspect ratio as physically possible. All the best flying and fastest birds have wings with a large aspect ratio, that is with a large span and small chord. The photographs of an albatross (Figure 3, left) and a starling (Figure 3, right) show examples, respectively, of high performance, long distance flight and of more manoevrable bird flight.

Figure 3: Photographs of an albatross (left) and a starling (right) in flight.

Also note for reference below that the mean lift, L, generated by this wing motion is simply the value of F_y averaged over one cycle of the oscillation or

$$L^* = \frac{L}{\frac{1}{2}\rho U^2 A_F} = \frac{1}{2\pi}\int_0^{2\pi}\left\{C_L + C_D(\frac{\omega h}{U})\cos\omega t\right\}\left\{1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right\}^{\frac{1}{2}} d(\omega t) \quad \text{(Dfe8)}$$

To evaluate the integrals in equations (Dfe7) and (Dfe8) it remains to specify C_L and C_D which, if we assume that the oscillatory motions are sufficiently slow that quasisteady coefficients can be used, will be functions primarily of the local angle of incidence, $\alpha^* = \theta - \alpha + \alpha_0$. For the purposes of this demonstration we will neglect the drag and assume the lift coefficient is the same as that of a flat plate namely

$$C_L \approx 2\pi \sin\alpha^* = 2\pi\sin\{\theta - \tilde{\alpha}\cos\omega t + \alpha_0\} \quad \text{(Dfe9)}$$

which becomes

$$C_L = 2\pi\left[\frac{\omega h}{U}\cos\omega t\cos\{\tilde{\alpha}\cos\omega t - \alpha_0\} - \sin\{\tilde{\alpha}\cos\omega t - \alpha_0\}\right]\left[1+\frac{\omega^2 h^2}{U^2}\cos^2\omega t\right]^{-\frac{1}{2}} \quad \text{(Dfe10)}$$

Substituting this into equation (Dfe7) and setting $C_D = 0$ yields

$$T^* = \frac{T}{\frac{1}{2}\rho U^2 A_F} = \frac{\omega h}{U}\int_0^{2\pi}\left[\frac{\omega h}{U}\cos\omega t\cos\{\tilde{\alpha}\cos\omega t - \alpha_0\} - \sin\{\tilde{\alpha}\cos\omega t - \alpha_0\}\right]\cos\omega t \; d(\omega t) \quad \text{(Dfe11)}$$

while substituting this into equation (Dfe8) and setting $C_D = 0$ yields

$$L^* = \frac{L}{\frac{1}{2}\rho U^2 A_F} = \int_0^{2\pi}\left[\frac{\omega h}{U}\cos\omega t\cos\{\tilde{\alpha}\cos\omega t - \alpha_0\} - \sin\{\tilde{\alpha}\cos\omega t - \alpha_0\}\right] d(\omega t) \quad \text{(Dfe12)}$$

Note that the dimensionless thrust, T^*, and lift, L^*, are functions only of the three dimensionless parameters, $\omega h/U$, $\tilde{\alpha}$ and α_0. If the angles $\tilde{\alpha}$ and α_0 are small the the above expressions may be approximately evaluated as

$$T^* \approx \pi \frac{\omega h}{U} \left\{ \frac{\omega h}{U} - \tilde{\alpha} \right\} \qquad \text{and} \qquad L^* \approx 2\pi\alpha_0 \tag{Dfe13}$$

so that the thrust is the same as in the fish locomotion case (unchanged by α_0) while the averaged dimensionless lift takes the expected value of $2\pi\alpha_0$. In level flight the lift, L, must balance the weight of the bird while the thrust, T, must balance the drag on the body of the bird and its wings.

One of the features in bird flight that is more important than for fish and cetaceans is that the instantaneous lift, F_y, oscillates in time and this could produce an unpleasant vertical oscillation of the body of the bird. Most birds are adapted to minimize this oscillation using a wing structure and musculature that generates a variation in the phase of the oscillation along the *span* of the wing in such a way that the total instantaneous lift transferred to the body has a minimal oscillatory component. In contrast most human-built ornithopters have rigid wings and tend to self-destruct under the oscillatory lift.

In the above analysis we have utilized the simple planar flow lift coefficient, $C_L = 2\pi\alpha$, and set $C_D = 0$. A more accurate analysis would utilize more precise lift and drag coefficients. Figure 4 presents some measured lift/drag polars for bird-like airfoils.

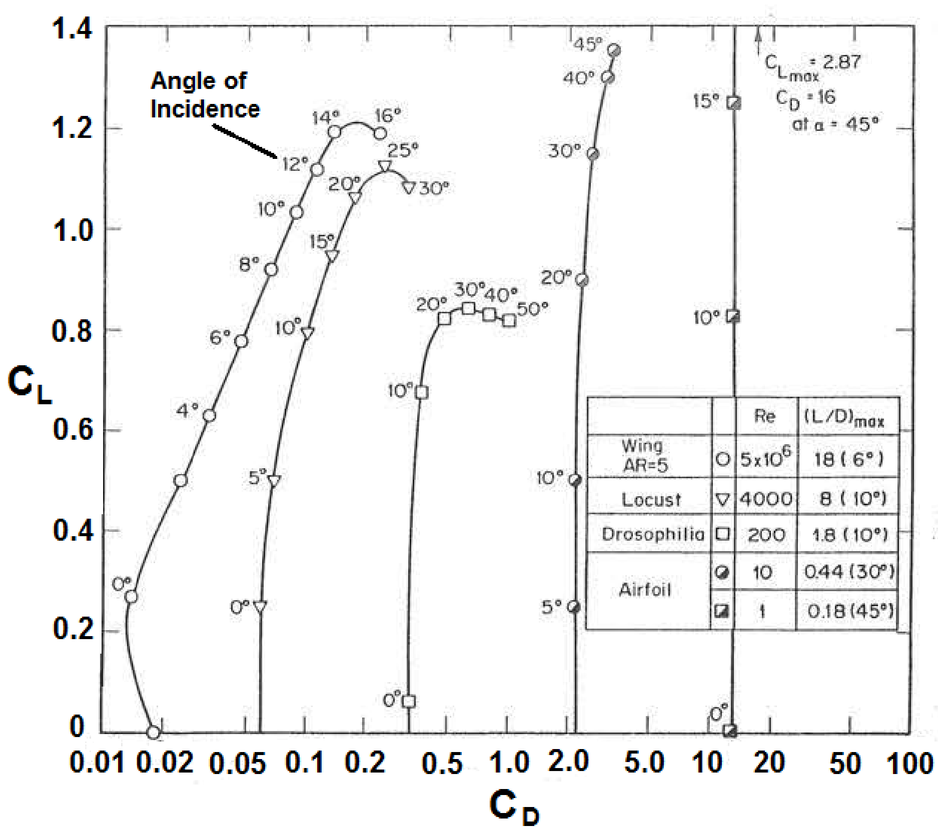

Figure 4: Lift/drag polars for a variety of wings. Adapted from Nachtigall (1974) and Thom and Swart (1940).