Compressibility

The compressibility of a fluid is a measure of how the density of that fluid changes with pressure. Consequently it is given by the derivative $(\partial\rho/\partial p)$; however, since the density is a thermodynamic quantity which is a function of two thermodynamic variables, this derivative can only be determined when some thermodynamic constraint is applied. Two common constraints are used, namely the constraint of constant temperature and the constraint of constant entropy. These lead to two commonly quoted compressibilities namely the isothermal and the isentropic compressibilities. Many of the fluid flows with which we will deal are sufficiently fast that there is relatively little time for the conduction of heat out of or into individual fluid elements. Consequently the isentropic compressibility is the most commonly applicable.

For most solids or liquids, the compressibility is usually quoted in terms of a bulk modulus of compressibility, κ, defined as

$$\kappa = \rho / \left(\frac{\partial\rho}{\partial p}\right)_Q \qquad \text{(Abc1)}$$

where the Q denotes the particular thermodynamic constraint discussed above (usually isentropic).

The bulk modulus of compressibility therefore has the units of pressure namely kg/ms^2 or Pa. It is useful to remember that water at normal temperatures and pressures has a bulk modulus of about $2206MPa$ and is therefore quite incompressible. Some other compressibility data is given in Table 1. Gases are, of course, much more compressible. For example, the isothermal compressibility of a **perfect gas** is readily evaluated by noting that in this instance $\partial\rho/\partial p = 1/(\mathcal{R}T)$ so that $\kappa = \rho\mathcal{R}T$ and κ is therefore simply equal to the pressure, p. On the other hand the more commonly pertinent adiabtic compressibility requires the adiabatic relation that $p \propto \rho^\gamma$ and differentiating this leads to $\kappa = \gamma p$. At normal pressures these bulk moduli are much smaller than than of a typical liquid.

TABLE 1. The bulk modulus of compressibility for several liquids (in MPa).

Liquid	Temperature	Bulk Modulus (MPa)
Water	$20°C$	2190
Carbon Tetrachloride	$20°C$	1390
Methanol	$20°C$	830
Gasoline	$20°C$	958
Linseed Oil	$20°C$	1600
SAE 30W Oil	$20°C$	1380
Mercury	$20°C$	25500