Energy Balance

The next step in the assessment of the performance of a turbomachine is to consider the application of the first and second laws of thermodynamics to such devices. In doing so we shall characterize the inlet and discharge flows by their pressure, velocity, enthalpy, etc., assuming that these are uniform flows. It is understood that when the inlet and discharge flows are non-uniform, the analysis actually applies to a single streamtube and the complete energy balance requires integration over all of the streamtubes.

The basic thermodynamic measure of the energy stored in a unit mass of flowing fluid is the total specific enthalpy (total enthalpy per unit mass) denoted by h^T and defined by

$$h^T = h + \frac{1}{2}|u|^2 + gz = e + \frac{p}{\rho} + \frac{1}{2}|u|^2 + gz \tag{Mbbf1}$$

where e is the specific internal energy, $|u|$ is the magnitude of the fluid velocity, and z is the vertical elevation. This expression omits any energy associated with additional external forces (for example, those due to a magnetic field), and assumes that the process is chemically inert.

Consider the steady state operation of a fluid machine in which the entering fluid has a total specific enthalpy of h_1^T, the discharging fluid has a total specific enthalpy of h_2^T, the mass flow rate is m, the net rate of heat addition to the machine is $\mathcal{Q}$, and the net rate of work done on the fluid in the machine by external means is $\dot{W}$. It follows from the first law of thermodynamics that

$$m(h_2^T - h_1^T) = \mathcal{Q} + \dot{W} \tag{Mbbf2}$$

Now consider incompressible, inviscid flow. It is a fundamental property of such a flow that it contains no mechanism for an exchange of thermal and mechanical energy, and, therefore, equation (Mbbf2) divides into two parts, governing the mechanical and thermal components of the total enthalpy, as follows

$$(p/\rho + \frac{1}{2}|u|^2 + gz)_2 - (p/\rho + \frac{1}{2}|u|^2 + gz)_1 = \frac{(p_2^T - p_1^T)}{\rho} = \frac{\dot{W}}{m} \tag{Mbbf3}$$

$$e_2 - e_1 = \mathcal{Q}/m \tag{Mbbf4}$$

Thus, for incompressible inviscid flow, the fluid mechanical problem (for which equation (Mbbf3) represents the basic energy balance) can be decoupled from the heat transfer problem (for which the heat balance is represented by equation (Mbbf4)).

It follows that, if T is the torque applied by the impeller to the fluid, then the rate of work done on the fluid is $\dot{W} = T\Omega$. Consequently, in the case of an ideal fluid which is incompressible and inviscid, equation (Mbbf3) yields a relation connecting the total pressure rise across the pump, $p_2^T - p_1^T$, the mass flow rate, m, and the torque:

$$m\frac{(p_2^T - p_1^T)}{\rho} = T\Omega \tag{Mbbf5}$$

Furthermore, the second law of thermodynamics implies that, in the presence of irreversible effects such as those caused by viscosity, the equality in equation (Mbbf5) should be replaced by an inequality, namely a "less than" sign. Consequently, in a real pump operating with an incompressible fluid, viscous effects will cause some of the input energy to be converted to heat rather than to an increase in the stored energy in the fluid. It follows that the right hand side of equation (Mbbf5) is the actual work done on the fluid by the impeller, and the left hand side is the fraction of that work which ends up as mechanical energy stored in the fluid. It is, therefore, appropriate to define a quantity, η_P, known as the pump hydraulic efficiency, to represent that fraction of the work done on the fluid that ends up as an increase in the mechanical energy stored in the fluid:

$$\eta_P = m\left(p_2^T - p_1^T\right)/\rho T\Omega \tag{Mbbf6}$$

Of course, additional mechanical losses may occur in a pump. These can cause the rate of work transmitted through the external shaft of the pump to be greater than the rate at which the impeller does work on the fluid. For example, losses may occur in the bearings or as a result of the "disk friction" losses caused by the fluid dynamic drag on other, non-active surfaces rotating with the shaft. Consequently, the overall (or shaft) efficiency, η_S, may be significantly smaller than η_P. For approximate evaluations of these additional losses, the reader is referred to the work of Balje (1981).

Despite all these loss mechanisms, pumps can be surprisingly efficient. A well designed centrifugal pump should have an overall efficiency in the neighborhood of 85% and some very large pumps (for example those in the Grand Coulee Dam) can exceed 90%. Even centrifugal pumps with quite simple and crude geometries can often be 60% efficient.