Notation

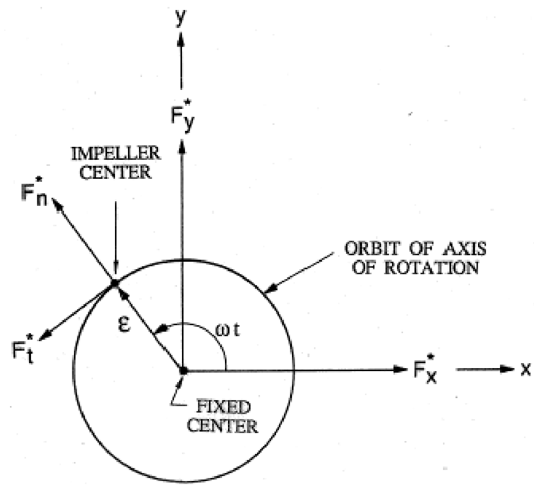

Figure 1: Schematic showing the relationship between the forces in the pump frame, F_x^*, F_y^*, the rotordynamic forces, F_n^*, F_t^*, the impeller center, the whirl orbit, and the volute geometry.

The forces that the fluid imparts to the rotor in a plane perpendicular to the axis of rotation are depicted in figure 1, and are decomposed into components in the directions x and y, where this coordinate system is fixed in the framework of the pump. The instantaneous forces are denoted by $F_x^*(t)$, $F_y^*(t)$, and the time-averaged values of these forces in the stationary frame are denoted by F_{0x}^*, F_{0y}^*. By definition, these are the steady forces commonly referred to as the radial forces or radial thrust. Sometimes it is important to know the axial position of the line of action of these forces. Alternatively, one can regard the x, y axes as fixed at some convenient axial location. Then, in addition to the forces, $F_x^*(t)$ and $F_y^*(t)$, the fluid-induced bending moments, $M_x^*(t)$ and $M_y^*(t)$, would be required information. The time-averaged moments will be defined by M_{0x}^* and M_{0y}^*.

Even if the location of the center of rotation were stationary at the origin of the xy plane (figure 1) the forces $F_x^*(t)$, $F_y^*(t)$ and moments $M_x^*(t)$, $M_y^*(t)$ could still have significant unsteady components. For example, rotor-stator interaction could lead to significant forces on the impeller at the blade passing frequencies. Similarly, there could be blade passing frequency components in the torque, $T(t)$, and the axial thrust, as discussed earlier in section (Mbfb). For simplicity, however, they will not be included in the present mathematical formulation.

The other set of forces with which this chapter will be concerned are the fluid-induced rotordynamic forces that are caused by the displacement and motion of the axis of rotation. It will be assumed that this displacement is sufficiently small so that a linear perturbation model is accurate. Then

$$\begin{Bmatrix} F_x^*(t) \\ F_y^*(t) \end{Bmatrix} = \begin{Bmatrix} F_{0x}^* \\ F_{0y}^* \end{Bmatrix} + [A^*] \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \qquad \text{(Mcb1)}$$

where the displacement is given by $x(t)$ and $y(t)$, and $[A^*]$ is known as the “rotordynamic force matrix,” which, in the linear model, would be independent of time, t. In virtually all cases that we shall be describing here, the displacements are sinusoidal. The “whirl” frequency of these motions will be denoted by ω (rad/s). Then, in general, the matrix $[A^*]$ will not only be a function of the turbomachine geometry and operating condition, but also of the whirl frequency, ω. In an analogous manner the rotordynamic moment matrix, $[B^*]$, is defined by

$$\begin{Bmatrix} M_x^*(t) \\ M_y^*(t) \end{Bmatrix} = \begin{Bmatrix} M_{0x}^* \\ M_{0y}^* \end{Bmatrix} + [B^*] \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} \tag{Mcb2}$$

The radial forces will be presented here in nondimensional form (denoted by the same symbols without the asterisk) by dividing the forces by $\rho\pi\Omega^2 R_{T2}^3 L$, where the selected length L may vary with the device. In seals and bearings, L is the axial length of the component. For centrifugal pumps, it is appropriate to use the width of the discharge so that $L = B_2$. With axial inducers, the axial extent of the blades is used for L. The displacements are nondimensionalized by R. In seals and bearings, the radius of the rotor is used; in centrifugal pump impellers, the discharge radius is used so that $R = R_{T2}$. It follows that the matrix $[A]$ is nondimensionalized by $\rho\pi\Omega^2 R^2 L$. Correspondingly, the radial moments and the moment matrix $[B]$ are nondimensionalized by $\rho\pi\Omega^2 R^4 L$ and $\rho\pi\Omega^2 R^3 L$ respectively. Thus

$$\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} = \begin{Bmatrix} F_{0x} \\ F_{0y} \end{Bmatrix} + [A] \begin{Bmatrix} x(t)/R \\ y(t)/R \end{Bmatrix} \tag{Mcb3}$$

$$\begin{Bmatrix} M_x(t) \\ M_y(t) \end{Bmatrix} = \begin{Bmatrix} M_{0x} \\ M_{0y} \end{Bmatrix} + [B] \begin{Bmatrix} x(t)/R \\ y(t)/R \end{Bmatrix} \tag{Mcb4}$$

The magnitude of the dimensionless radial force will be denoted by $F_0 = (F_{0x}^2 + F_{0y}^2)^{\frac{1}{2}}$, and its direction, θ, will be measured from the tongue or cutwater of the volute in the direction of rotation.

One particular feature of the rotordynamic matrices, $[A]$ and $[B]$, deserves special note. There are many geometries in which the rotordynamic forces should be invariant to a rotation of the x, y axes. Such will be the case only if

$$A_{xx} = A_{yy} \quad ; \quad A_{xy} = -A_{yx} \tag{Mcb5}$$

$$B_{xx} = B_{yy} \quad ; \quad B_{xy} = -B_{yx} \tag{Mcb6}$$

This does appear to be the case for virtually all of the experimental measurements that have been made in turbomachines.

The prototypical displacement will clearly consist of a circular whirl motion of "eccentricity", ϵ, and whirl frequency, ω, so that $x(t) = \epsilon \cos \omega t$ and $y(t) = \epsilon \sin \omega t$. As indicated in figure 1, an alternative notation is to define "rotordynamic forces", F_n^* and F_t^*, that are normal and tangential to the circular whirl orbit at the instantaneous position of the center of rotation. Note that F_n^* is defined as positive outward and F_t^* as positive in the direction of rotation, Ω. It follows that

$$F_n^* = \epsilon \left(A_{xx}^* + A_{yy}^* \right) /2 \tag{Mcb7}$$

$$F_t^* = \epsilon \left(A_{yx}^* - A_{xy}^* \right) /2 \tag{Mcb8}$$

and it is appropriate to define dimensionless normal and tangential forces, F_n and F_t, by dividing by $\rho\pi\Omega^2 R^2 L\epsilon$. Then the conditions of rotational invariance can be restated as

$$A_{xx} = A_{yy} = F_n \tag{Mcb9}$$

$$A_{yx} = -A_{xy} = F_t \tag{Mcb10}$$

Since this condition is met in most of the experimental data, it becomes convenient to display the rotordynamic forces by plotting F_n and F_t as functions of the geometry, operating condition and frequency ratio, ω/Ω. This presentation of the rotordynamic forces has a number of advantages from the perspective of physical interpretation. In many applications the normal force, F_n, is modest compared with the potential restoring forces which can be generated by the bearings and the casing. The tangential force has greater significance for the stability of the rotor system. Clearly a tangential force that is in the same direction as the whirl velocity ($F_t > 0$ for $\omega > 0$ or $F_t < 0$ for $\omega < 0$) will be rotordynamically destabilizing, and will cause a fluid-induced reduction in the critical whirl speeds of the machine. On the other hand, an F_t in the opposite direction to ω will be whirl stabilizing.

Furthermore, it is conventional among rotordynamicists to decompose the matrix $[A]$ into added mass, damping and stiffness matrices according to

$$[A]\begin{Bmatrix} x/R \\ y/R \end{Bmatrix} = -\begin{bmatrix} M & m \\ -m & M \end{bmatrix}\begin{Bmatrix} \ddot{x}/R\Omega^2 \\ \ddot{y}/R\Omega^2 \end{Bmatrix} - \begin{bmatrix} C & c \\ -c & C \end{bmatrix}\begin{Bmatrix} \dot{x}/R\Omega \\ \dot{y}/R\Omega \end{Bmatrix} - \begin{bmatrix} K & k \\ -k & K \end{bmatrix}\begin{Bmatrix} x/R \\ y/R \end{Bmatrix} \tag{Mcb11}$$

where the dot denotes differentiation with respect to time, so that the added mass matrix, $[M]$, multiplies the acceleration vector, the damping matrix, $[C]$, multiplies the velocity vector, and the stiffness matrix, $[K]$, multiplies the displacement vector. Note that the above has assumed rotational invariance of $[A]$, $[M]$, $[C]$ and $[K]$; M and m are respectively termed the direct and cross-coupled added mass, C and c the direct and cross-coupled damping, and K and k the direct and cross-coupled stiffness. Note also that the corresponding dimensional rotordynamic coefficients, M^*, m^*, C^*, c^*, K^*, and k^* are related to the dimensionless versions by

$$M, m = \frac{M^*, m^*}{\rho\pi R^2 L} \quad ; \quad C, c = \frac{C^*, c^*}{\rho\pi R^2 L\Omega} \quad ; \quad K, k = \frac{K^*, k^*}{\rho\pi R^2 L\Omega^2} \tag{Mcb12}$$

The representation of equation (Mcb11) is equivalent to assuming a quadratic dependence of the elements of $[A]$ (and the forces F_n, F_t) on the whirl frequency, or frequency ratio, ω/Ω. It should be emphasized that fluid mechanical forces do not always conform to such a simple frequency dependence. For example, in section (Nlg), we shall encounter a force proportional to $\omega^{\frac{3}{2}}$. Nevertheless, it is of value to the rotordynamicists to fit quadratics to the plots of F_n and F_t against ω/Ω, since, from the above relations, it follows that

$$F_n = M\left(\omega/\Omega\right)^2 - c\left(\omega/\Omega\right) - K \tag{Mcb13}$$

$$F_t = -m\left(\omega/\Omega\right)^2 - C\left(\omega/\Omega\right) + k \tag{Mcb14}$$

and, therefore, all six rotordynamic coefficients can be directly evaluated from quadratic curve fits to the graphs of F_n and F_t against ω/Ω.

Since m is often small and is frequently assumed to be negligible, the sign of the tangential force is approximately determined by the quantity $k\Omega/\omega C$. Thus rotordynamicists often seek to examine the quantity $k/C = k^*/\Omega C^*$, which is often called the "whirl ratio" (not to be confused with the whirl frequency ratio, ω/Ω). Clearly larger values of this whirl ratio imply a larger range of frequencies for which the tangential force is destabilizing and a greater chance of rotordynamic instability.

In the last few paragraphs we have focused on the forces, but it is clear that a parallel construct is relevant to the rotordynamic moments. It should be recognized that each of the components of a turbomachine will manifest its own rotordynamic coefficients which will all need to be included in order to effect a complete rotordynamic analysis of the machine. The methods used in such rotordynamic analyses are beyond the scope of this book. However, we shall attempt to review the origin of these forces in the bearings, seals, and other components of the turbomachine. Moreover, both the main flow and leakage flows associated with the impeller will generate contributions. In order to permit ease of comparison between the rotordynamic effects contributed by the various components, we shall use a similar nondimensionalization for all the components.