Impulse and Reaction Turbines

Impulse and reaction turbines evolved from the Pelton wheel turbine. The impulse turbine was similar except that the buckets of the Pelton wheel were replaced by streamline blades as depicted in Figure 1. These are driven by a jet that impacts the blades from one side and the reflected stream emerges from the other. The reaction turbine is similar except that there is usually incident flow all around the periphery whose direction is determined by a cascade or set of inlet guide vanes. The performance of an impulse or

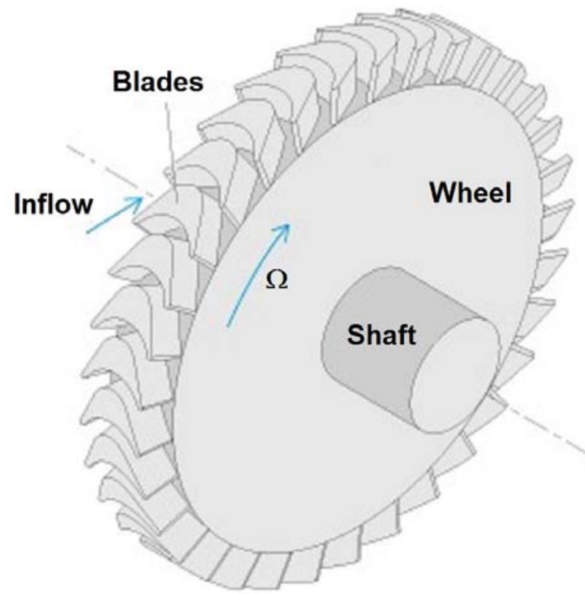

Figure 1: Sketch of a single stage impulse or reaction turbine rotor.

reaction turbine can be assessed by the same analysis as follows. As depicted in Figure 2, this incident jet or flow of velocity, V, results in a force driving the turbine with a peripheral speed, U. In order to apply Bernoulli's equation and the momentum theorem to the flow through a set of blades, it is necessary to view the flow in a frame of reference in which the blades are at rest and the flow is steady. In such a relative frame, since both the entering and exiting jets are at the pressure of the containment vessel, then by Bernoulli's equation they would have the same velocity magnitude in the absence of gravity and viscous, frictional effects. In order to take such viscous effects into account in the present analysis it is assumed that these can be represented by the constant, C, where the relative velocity leaving blades is $-C$ times the relative velocity entering the blades and C is a little less than unity. We will also denote the mass flow rate through all stages in the direction perpendicular to U by m. It is convenient to assume that all the angles α and β of the flows entering and leaving the blades are sufficiently small so that $\cos\alpha$ and $\cos\beta$ can be approximated by unity.

We begin with an analysis of the simplest impulse turbine consisting of a single set of rotor blades as depicted in Figure 2. The velocities entering the rotor are denoted by the subscript 1 while those leaving are denoted by the subscript 2. In order to use the steady flow version of the momentum theorem we must utilize a control volume around the rotor which is moving with the rotor at the velocity U. Then the

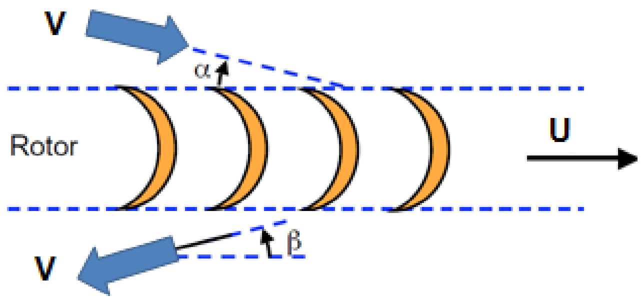

Figure 2: Single stage of an impulse turbine.

velocities relative to that control volume in the U direction are approximately:

$$u_1 = V \qquad \text{and} \qquad v_1 = V - U \qquad \text{and} \qquad v_2 = -Cv_1 = -C(V-U)$$

The net momentum flux in the U direction exiting the rotor is therefore

$$m(v_2 - v_1) = -m(V-U)(1+C)$$

and by the momentum theorem this must be equal to the force on the fluid in the U direction within the rotor. Consequently the force on the rotor in the U direction, F_{R1}, is

$$F_{R1} = m(V-U)(1+C)$$

It is conventional to define a **blade efficiency**, η_{1rotor}, as the ratio of the power transmitted to the rotor, $F_{R1}U$, to the available energy in the incoming flow, $mV^2/2$. For this single rotor impulse turbine the blade efficiency is therefore

$$\eta_{1rotor} = \frac{2F_{R1}U}{mV^2} = \frac{2U}{V}(1+C)\left\{1 - \frac{U}{V}\right\}$$

For a representative value of C equal to 0.9 this becomes

$$\eta_{1rotor} = \frac{2U}{V}\left[1.9 - 1.9\frac{U}{V}\right]$$

and we use this result below to compare turbines with various numbers of rotors.

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Reaction turbines often consist of multiple stages in order to take full advantage of all the energy in the inlet flow. A two stage reaction turbine is depicted in Figure 3 and consists of a rotor followed by a stator followed by a second rotor. The analysis begins with the same results for the first rotor as were detailed above for the single rotor turbine. The velocities entering and leaving the subsequent stator will be denoted by subscripts 3 and 4 respectively. Since the velocities relative to the stator are identical to those in the non-rotating frame of reference it follows that

$$v_3 = u_3 = v_2 + U = U(1+C) - CV \qquad \text{and} \qquad v_4 = u_4 = -Cu_3 = C^2V - UC(1+C)$$

Now we turn attention to the second rotor. As with the first rotor, in order to use the steady flow version of the momentum theorem, we must utilize a control volume around the second rotor which is moving with

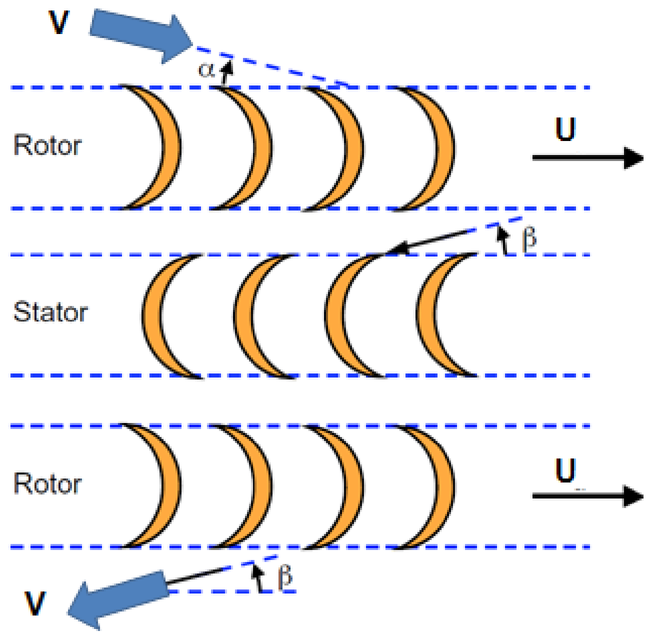

Figure 3: A two-stage reaction turbine.

the second rotor at the velocity U. The velocities relative to that control volume in the U direction at the entrance to and exit from the second rotor are denoted by subscripts 5 and 6 and are approximately

$$v_5 = u_4 - U = -Cu_3 = C^2V - U(1 + C + C^2) \qquad \text{and} \qquad v_6 = -Cv_5 = -C^3V + UC(1 + C + C^2)$$

The net momentum flux in the U direction exiting the second rotor is therefore

$$m(v_6 - v_5) = -m[VC^2(1 + C) - U(1 + 2C + 2C^2 + C^3)]$$

and this must be equal to the force on the fluid in the U direction within the second rotor. Consequently the force on the second rotor in the U direction, F_{R2}, is

$$F_{R2} = m[VC^2(1 + C) - U(1 + 2C + 2C^2 + C^3)]$$

Adding F_{R1} and F_{R2} the total force, F_R, on the rotors in the U direction is therefore

$$F_R = m[VC^2(1+C)-U(1+2C+2C^2+C^3)+(V-U)(1+C)] = m[V(1+C+C^2+C^3)-U(2+3C+2C^2+C^3)]$$

Consequently the power, P, transmitted to the rotor is $P = F_RU$. The blade efficiency of this two stage impulse turbine, η_{2rotor}, is defined as the ratio of this power to the available energy in the incoming flow prior to the first stage namely $mV^2/2$ so that the blade efficiency in this case is given by

$$\eta_{2rotor} = \frac{2U}{V}\left[(1 + C + C^2 + C^3) - (2 + 3C + 2C^2 + C^3)\frac{U}{V}\right]$$

For example for a value of C equal to 0.9 this becomes

$$\eta_{2rotor} = \frac{2U}{V}\left[3.239 - 6.849\frac{U}{V}\right]$$

This is a maximum when $U/V = 3.239/(2 \times 6.849) = 0.236$.

To add more stages we could simply continue the analysis detailed above. A second stator that followed the second rotor would have entering and leaving velocities denoted by the subscripts 7 and 8 given by

$$v_7 = u_7 = v_6 + U = -C^3V + U[1+C+C^2+C^3] \quad \text{and} \quad v_8 = u_8 = -Cu_7 = C^4V - UC[1+C+C^2+C^3]$$

and these mean that the relative velocities entering (subscript 9) and leaving (subscript 10) a third rotor would be

$$v_9 = u_8 - U = C^4V - U[1+C+C^2+C^3+C^4] \quad \text{and} \quad v_{10} = -Cv_9 = -C^5V + UC[1+C+C^2+C^3+C^4]$$

so that the net momentum flux in the U direction exiting a third rotor would be

$$m(v_{10} - v_9) = -m[VC^2(1+C) - U(1+2C+2C^2+C^3)]$$

This would be equal to the force on the fluid in the U direction within the third rotor. Consequently the force on the third rotor in the U direction, F_{R3}, would be

$$F_{R3} = m[VC^4(1+C) - U(1+2C+2C^2+2C^3+2C^4+C^5)]$$

and the total force on a three stage rotor then becomes

$$F_R = m[V(1+C+C^2+C^3+C^4+C^5) - U(3+5C+4C^2+3C^3+2C^4+C^5)]$$

Therefore the blade efficiency for a three stage reaction turbine, η_{3rotor}, becomes

$$\eta_{3rotor} = \frac{2U}{V}\left[(1+C+C^2+C^3+C^4+C^5) - (3+5C+4C^2+3C^3+2C^4+C^5)\frac{U}{V}\right]$$

which, for a value of C equal to 0.9, is

$$\eta_{3rotor} = \frac{2U}{V}\left[4.69 - 14.83\frac{U}{V}\right]$$

Comparing the one, two and three rotor turbine blade efficiencies, for example for $U/V = 0.1$, we find $\eta_{1rotor} = 0.342$, $\eta_{2rotor} = 0.511$, and $\eta_{3rotor} = 0.642$ and therefore the blade efficiciences increase as more rotors extract more energy from the flow.

A more appropriate comparison would be to examine the maximum blade efficiencies for each of these turbines. For this purpose we differentiate the expressions for η_{1rotor}, η_{2rotor}, and η_{3rotor} with respect to U/V and then set those expressions to zero to find the values of U/V at which the blade efficiencies for each of the one, two and three stage turbines are a maximum. Then we can evaluate the blade efficiencies at those values of U/V. In the case of $C = 0.9$ this leads to the following results:

	$(U/V)_{max}$	η_{max}
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One Rotor Reaction Turbine	0.500	0.95
Two Rotor Reaction Turbine	0.236	0.766
Three Rotor Reaction Turbine	0.158	0.742

Consequently the lighter the load on the turbine (the larger the value of U/V) the fewer the number of stages needed and the higher the blade efficiency. On the other hand for larger loads and lower U/V the greater the number of stages needed to extract the energy from the inlet flow.