Capillarity

As we have seen a **meniscus** in a narrow gap between two plates or in a small diameter tube can be significantly elevated or depressed by the combined effects of **surface tension** and **contact angle**. Here we explore further this phenomenon of **capillarity**. Consider the meniscus in a small diameter cylindrical tube partially immersed in a large bath of liquid as sketched in Figure 1: If the tube radius, R, is sufficiently

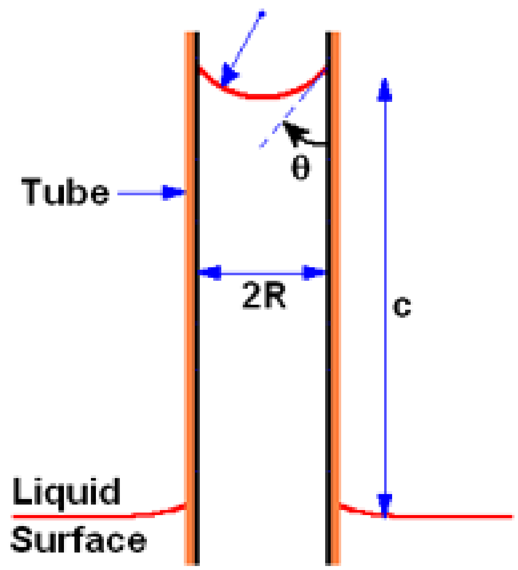

Figure 1: A meniscus in a capillary tube of internal radius, R.

small compared with the elevation or depression of the meniscus, c, then the pressure, p_L, in the liquid immediately beneath the liquid surface can be assumed constant across the meniscus so that the pressure difference with the atmospheric pressure, p_A, above the surface is uniform across the meniscus, This, in turn, implies that the curvature of the meniscus is uniform and therefore that the meniscus is spherically-shaped. It then follows that if the contact angle with the walls of the tube is θ, geometry dictates that the radius of curvature of the meniscus must be equal to $R/\cos\theta$ and the pressure difference must be given by

$$p_A - p_L = 2S\cos\theta/R \tag{Cq1}$$

But the pressure p_L in the liquid under the meniscus must also be given by $p_A - \rho g c$ and so

$$c = \frac{2S\cos\theta}{\rho g R} \tag{Cq2}$$

This is the commonly used formula for evaluating the capillary height, c, the elevation or depression of the meniscus in a narrow capillary tube. Note that when the contact angle is less than $\pi/2$ (for example for water and glass or metal) c is positive and the meniscus is elevated within the tube. On the other hand when the angle is greater than $\pi/2$ (for example for mercury and glass) the meniscus is depressed. Note also that the height, c, increases with the surface tension and it increases when the tube radius, R, is decreased. Thus the effect is most pronounced with very small capillary tubes.

It is instructive to describe another equivalent derivation of the formula for the capillary height. Consider the vertical forces acting on the volume of liquid inside the tube and above the external waterline. Since the pressure at the base of this volume is atmospheric that upward force must be balanced by the downward force due to atmospheric pressure acting on the top of this volume. The only remaining forces which must therefore balance are: (1) the weight equal to $\rho g \pi R^2 c$ and (2) the upward component of the surface tension force acting at the junction of the meniscus with the tube wall, namely $2\pi RS\cos\theta$. Equating these forces leads to the same equation (Cq2) for the capillary height, c. This second method of derivation may be more convenient in cases of more complex tube and meniscus geometries.

There are many important technological and natural phenomena in which capillarity is important. For example, a wick uses the fine holes in its fabric to draw the liquid up from the reservoir below. Similarly a leaf draws the sap up from the ground using the fine pores in its surface. Indeed this latter phenomena caused botanists and other scientists much puzzlement in the past because, they argued, the pressure in the liquid column below the meniscus must necessarily be below atmospheric. Consequently no tree or plant should be able to grow higher than about $10m$ for otherwise that pressure would fall below the vapor pressure and surely the sap would then vaporize. Some investigators placed stethoscopes against trees trying to hear the sound of the sap cavitating on a hot day. Others attempted to tap into trees to measure the pressure in the liquid column. One explanation offered was that sap was so pure that it could withstand negative pressures of tens of atmospheres. The truth seems to be that tall trees have other internal mechanisms that propel the sap upwards and do not rely solely on the capillary effect of the leaves.