Fluid static forces

In this section we derive some basic results for the forces on bounding walls or solid surfaces as a result of the pressure, p, acting on that surface. Consider a small piece of solid surface of area, δA, in contact with a fluid and at an arbitrary inclination relative to a coordinate framework (x, y, z). If we denote the outward unit vector normal to the surface by $\underline{n}$ then clearly the vector force acting on that surface is $pdA\underline{n}$. It is usually more convenient to consider the components of this force in the x, y and z directions namely $pdA\ n_x$, $pdA\ n_y$ and $pdA\ n_z$ respectively. However it is convenient to note that the terms $dA\ n_x$, $dA\ n_y$ and $dA\ n_z$ in these expressions are simply the projected areas in the x, y and z directions. Therefore, the component in any arbitrary direction of the fluid force on a bounding wall or solid surface due to a pressure, p, is simply that pressure multiplied by the projected area of the surface in that arbitrary direction. This simple result can be very convenient when dealing with complicated curved surfaces.

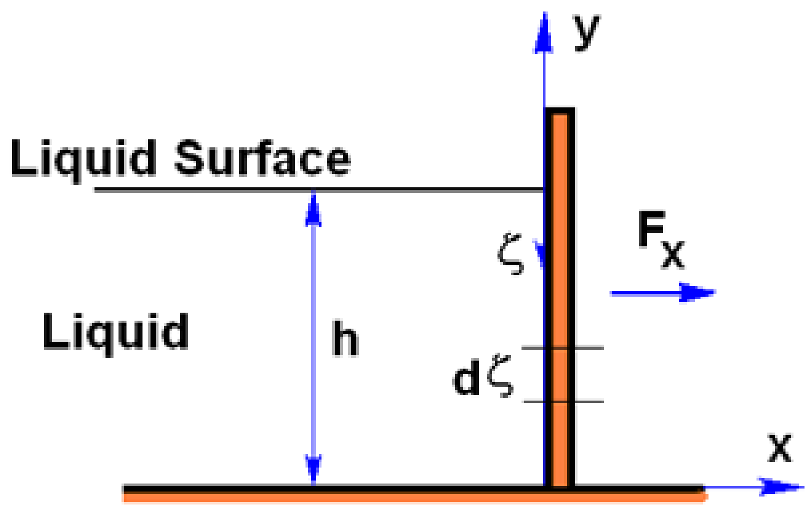

Figure 1: Sketch of a two-dimensional dam and reservoir, depth h.

As a example, consider the horizontal force imposed on the sidewall containing a large body of still liquid as sketched in figure 1. This could be the side of a swimming pool, a dam or any containing surface. For simplicity we will assume two-dimensional geometry and that the breadth perpendicular to the sketch is b. If, for convenience, we define a vertically downward coordinate, ζ (so that $\zeta = -y$), with its origin at the surface of the liquid, then the pressure at any depth in the liquid is $p_a + \rho g \zeta$ where p_a is the atmospheric pressure (ρg is assumed constant). Consequently the horizontal force imposed on the dam by the element, $d\zeta$, is $(p_a + \rho g \zeta) b d\zeta$. Partly balancing this is the force on the same element imposed by the atmospheric pressure on the "dry" side of the dam which is $p_a b d\zeta$ so that the net horizontal force on the dam due to the element, $d\zeta$, is dF_x where

$$dF_x = \rho g b \zeta d\zeta \qquad \text{(Cf1)}$$

To obtain the total horizontal force, F_x, we integrate this expression from $\zeta = 0$ to $\zeta = h$ and obtain:

$$F_x = \frac{1}{2} \rho g b h^2 \qquad \text{(Cf2)}$$

Note that these forces on the sides of liquid tanks and reservoirs can be very large. For example a typical swimming pool might have a depth, $h = 3\ m$ and a breadth, $b = 25\ m$. Equation (Cf2) then yields a force of $1.1 \times 10^6\ kg\ m/s^2$, equivalent to the weight of over 100 metric tons. Much more impressive is the force on a large dam. For example a dam with a depth of 300 m and a breadth of 300 m would experience a horizontal force of 13 million metric tons.

In addition to the magnitude of the force, F_x, it is important to know the line of action of this force so that the moments on the dam can be calculated. To obtain the location of the line of action we begin by denoting its vertical location by ζ_m. Then the force, dF_x, on the element, $d\zeta$, would have an anticlockwise moment, dM, about the location, $\zeta = \zeta_m$, given by

$$dM = \rho g b \zeta(\zeta - \zeta_m) d\zeta \tag{Cf3}$$

Integrating this expression from $\zeta = 0$ to $\zeta = h$ we obtain the total fluid induced moment, M, about the chosen location ζ_m:

$$M = \rho g b \left[\frac{h^3}{3} - \frac{\zeta_m h^2}{2} \right] \tag{Cf4}$$

Then the line of action of the force F_x is at the location ζ_m for which this moment, M, is zero. Hence

$$\zeta_m = \frac{2h}{3} \tag{Cf5}$$

and, as might have been anticipated, the force acts at 2/3 of the depth.