Acoustic Doppler Velocimetry

An Acoustic Doppler Velocimeter utilizes the fact that the velocity of a flow alters the velocity of sound along the direction of flow. In most of the practical instruments that utilize this principle to measure velocity, an acoustic wave is propagated from a ransmitter to one or two receivers at different positions relative to the direction of flow. Then cross-correlation of the signals from the receivers is used to obtain a time delay between the signals and a flow velocity is derived from that delay time. Acoustic Doppler Velocimeters come in a number of geometric configurations of Figure 1 and Figure 2 are two examples. These yield some "average" value of the fluid velocity or flow rate between two transceivers on opposites sides of the duct containing the flow. Figure 1 depicts the configuration of an ultrasonic flowmeter used by NASA in a cryogenic rocket engine

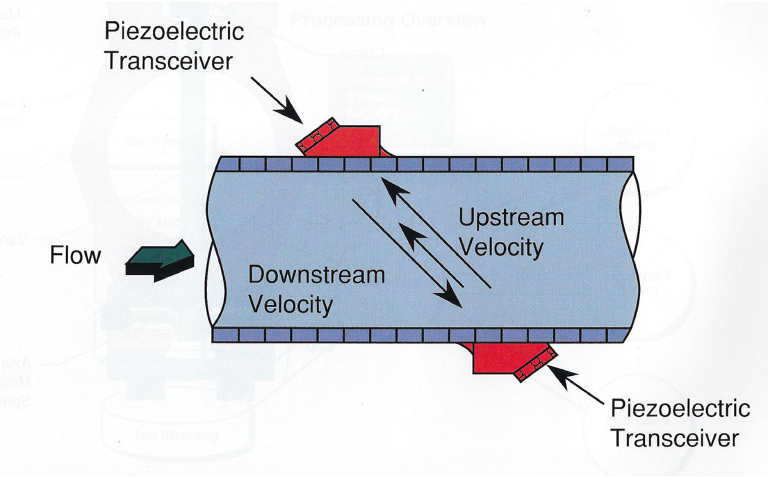

Figure 1: Schematic of a cryogenic Ultrasonic Flowmeter.

Another typical configuration is shown diagrammatically in Figure 2 and we demonstrate the basic principles using this configuration. An acoustic transmitter or source is placed in one wall and two acoustic receivers are placed in the opposite wall, one a distance, b, upstream of the source and the other an equal distance, b, downstream of the source. Then, if the acoustic speed in the fluid is denoted by c and the fluid velocity (assumed uniform everywhere) is denoted by U, the time delay, t_1, between the source and

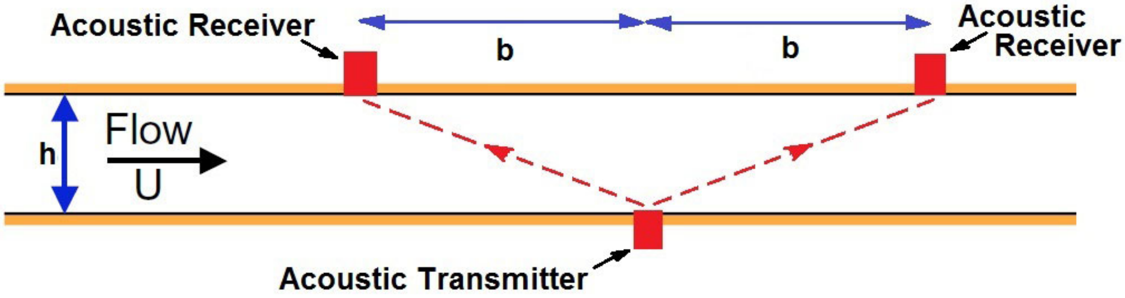

Figure 2: Schematic of a typical Acoustic Doppler Velocimeter.

the upstream receiver will be given by

$$t_1 = \frac{1}{c}\left[h^2 + (b+Ut_1)^2\right]^{\frac{1}{2}} \tag{Kdce1}$$

On the other hand the time delay, t_2, between the source and the downstream receiver will be given by

$$t_2 = \frac{1}{c}\left[h^2 + (b-Ut_2)^2\right]^{\frac{1}{2}} \tag{Kdce2}$$

It follows that the time difference between the arrival times of the signals at the two receivers is given by

$$t_1 - t_2 = \frac{2bU}{(c^2-U^2)} \tag{Kdce3}$$

and therefore the velocity, U, can be obtained by measuring the time delay or Doppler shift between the signals received by the two receiving transducers.

To explore the effect of a non-uniform stream, $u(y)$ (coordinate, y, is measured normal to the transmitter wall which is at $y = 0$, coordinate x is measured along the transmitter wall in the downstream direction and the velocity u is in the x direction), we examine a small increment, $dx \times dy$, of the ray from the transmitter to the upstream receiver. The time of transit, dt_1, of a sonic wave diagonally across that increment is given by

$$dt_1 = \frac{1}{c}\left[(dy)^2 + \left\{\frac{b\;dy}{h} + u\;dt_1\right\}^2\right]^{\frac{1}{2}} \tag{Kdce4}$$

which, upon rearrangement, yields

$$\frac{dt_1}{dy} = \frac{1}{(c^2-u^2)}\left\{\frac{bu}{h} + \left[\frac{b^2u^2}{h^2} + (c^2-u^2)\left(1+\frac{b^2}{h^2}\right)\right]^{\frac{1}{2}}\right\} \tag{Kdce5}$$

and with a similar expression for the same downstream increment on the ray between the transmitted and the downstream receiver the difference between the transit times, $dt_1 - dt_2 = \Delta t$, for the upstream and downstream increments at a distance y from the lower surface is given by

$$\frac{d\Delta t}{dy} = \frac{2bu(y)}{h(c^2-u^2)} \tag{Kdce6}$$

which is clearly in accord with the previous expression (Kdce3). Integrating over the channel, it follows that the total transit time difference between the transmitter and the receivers, Δt, is given by

$$\Delta t = \int_0^h \frac{2bu(y)}{h(c^2-u^2)}\;dy \tag{Kdce7}$$

To simplify matters we will assume that $u \ll c$ and therefore

$$\Delta t = \frac{2b}{c^2}\overline{u} \qquad \text{where} \qquad \overline{u} = \frac{1}{h}\int_0^h u(y)\;dy \tag{Kdce8}$$

and $\overline{u}$ is an average fluid velocity in the flow. Since equation (Kdce8) is essentially identical to equation (Kdce3) we can conclude that a non-uniform velocity profile has little or no effect on the calibration of the simple acoustic Doppler velocimeter.

This acoustic Doppler principle has been substantially expanded to create three dimensional external velocimeters such as that marketed by SonTek for use in water flows. That device, which utilizes the phase shift from scattered sound, is depicted diagrammatically in Figure 3. A central, focussed acoustic transmitter bombards a small measuring volume with sound which scatters sound toward three surrounding, focussed acoustic receivers. The doppler phase shifts in the sound detected by those receivers is then used to determine the average velocities in the flow between the measuring volume and the receivers.

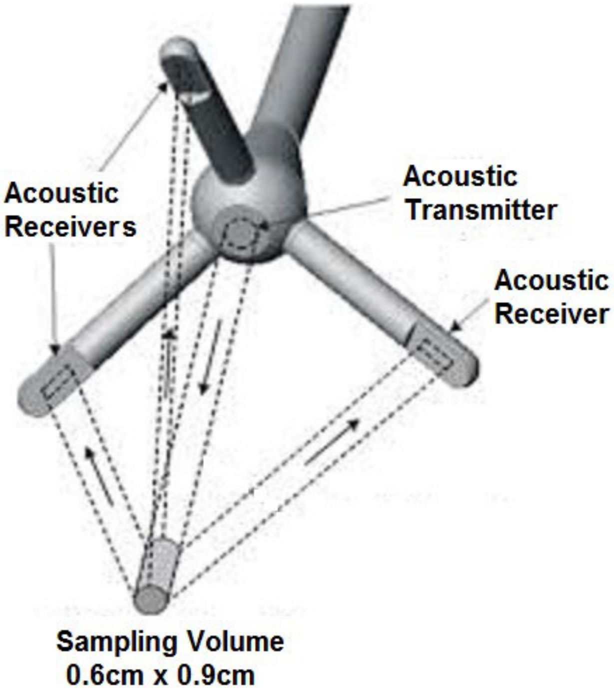

Figure 3: Schematic of the three-dimensional Acoustic Doppler Velocimeter marketed by SonTek.