Force Balances

As indicated in the last section, force balances and their design tend to be very specific to the task in hand and it is therefore not possible to provide too much in the way of general guidance. However, an example can provide some guidance. We therefore describe a force balance designed to measure both the steady and unsteady forces and moments communicated between a rotating shaft and an impeller (or propeller). The balance mounts onto the drive shaft and impeller, in turn, mounts onto the balance. The device is designed to operate in water and the strain gauges used therefore need to be contained in a watertight container.

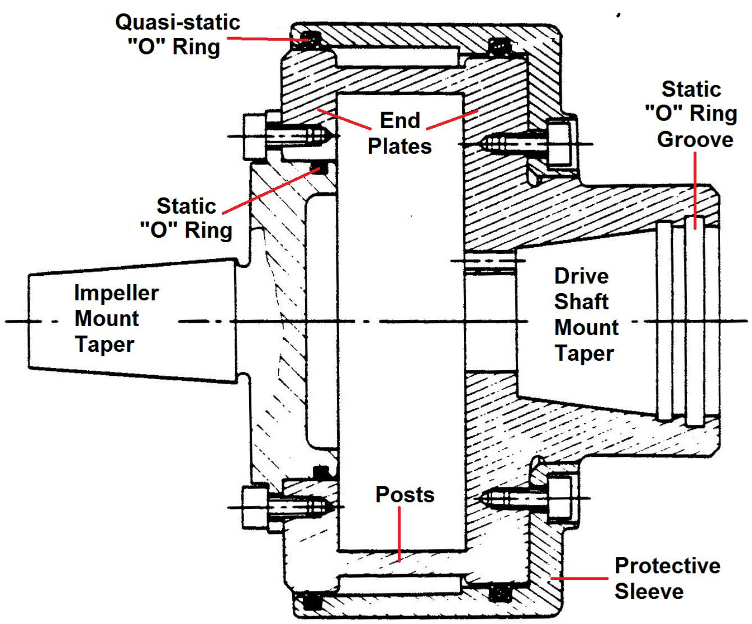

Figure 1: A force balance for a rotating shaft.

A four-post, rotating force balance designed to measure the shear forces, F_1 and F_2, bending moments, M_1 and M_2, torque, T, and axial thrust, P, is depicted in Figures 1, 2 and 3. The length of the posts is denoted by L, their radial location by R and the square cross-section dimension by a; the dimensionless parameter, $\lambda = L/a$ and the Young's modulus of the metal is denoted by E. Then if the strains registered by each of the strain gauge rosettes are denoted by ϵ_{Xki}, $X = A, B, C, D$, $k = 1-4$, $i = 1-2$, it follows that the forces and moments are given by

$$F_1 = \frac{Ea^3}{6\lambda}\left[(\epsilon_{A22} - \epsilon_{A21} - \epsilon_{A42} + \epsilon_{A41}) - (\epsilon_{C22} - \epsilon_{C21} - \epsilon_{C42} + \epsilon_{C41}) + (\epsilon_{B12} - \epsilon_{B11} - \epsilon_{B32} + \epsilon_{B31}) - (\epsilon_{D12} - \epsilon_{D11} - \epsilon_{D32} + \epsilon_{D31})\right] \quad \text{(Kdeb1)}$$

$$F_2 = \frac{Ea^3}{6\lambda}\left[(\epsilon_{A11} - \epsilon_{A12} - \epsilon_{A31} + \epsilon_{A32}) - (\epsilon_{C11} - \epsilon_{C12} - \epsilon_{C31} + \epsilon_{C32}) + (\epsilon_{B22} - \epsilon_{B21} - \epsilon_{B42} + \epsilon_{B41}) - (\epsilon_{D22} - \epsilon_{D21} - \epsilon_{D42} + \epsilon_{D41})\right] \quad \text{(Kdeb2)}$$

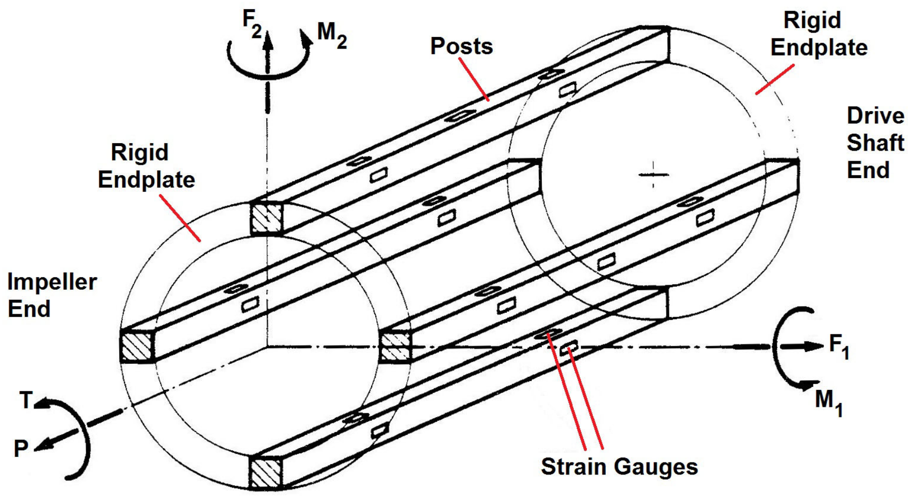

Figure 2: Rotating force balance consisting of four posts connecting two end plates, all machined out of a monolithic metal block. The posts are instrumented with strain gauges to measure the forces and moments experienced by the impeller and defined as acting on the impeller end of the force balance.

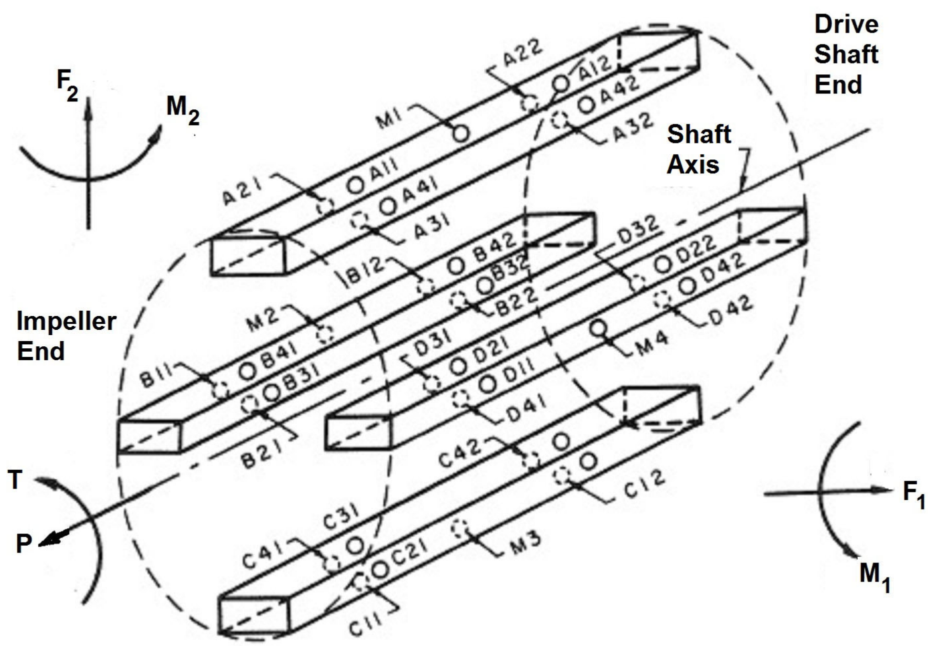

Figure 3: As Figure 2 showing the four posts, $X = A, B, C,$ and D, each instrumented with 9 strain gauge rosettes, four at quarter-length ($Xk1$, $k = 1-4$), one at mid-length (Mk, $k = 1-4$) and four at three-quarter length ($Xk2$, $k = 1-4$).

$$P = \frac{Ea^2}{4}\sum_{k=1}^{4} \epsilon_{Mk} \tag{Kdeb3}$$

$$M_1 = \frac{Ea^2R}{2}\left\{\frac{1}{4} + \frac{a^2}{48R^2}\right\}\sum_{k=1}^{4}\left\{\sum_{i=1}^{2}(\epsilon_{Aki} - \epsilon_{Cki})\right\} \tag{Kdeb4}$$

$$M_2 = \frac{Ea^2R}{2}\left\{\frac{1}{4} + \frac{a^2}{48R^2}\right\}\sum_{k=1}^{4}\left\{\sum_{i=1}^{2}(\epsilon_{Bki} - \epsilon_{Dki})\right\} \tag{Kdeb5}$$

$$T = \frac{Ea^3R}{6\lambda} \sum_{X=A}^{D} \{\epsilon_{X21} - \epsilon_{X22} - \epsilon_{X41} + \epsilon_{X42}\} \tag{Kdeb6}$$

Typical strain magnitudes, ϵ_F, ϵ_P, ϵ_M and ϵ_T associated with the shear forces, thrust, bending moments and torque are

$$\epsilon_F = \frac{3\lambda F}{8Ea^2} \quad ; \quad \epsilon_P = \frac{P}{4Ea^2} \quad ; \quad \epsilon_M = \frac{M}{8Ea^2R} \quad ; \quad \epsilon_T = \frac{3\lambda T}{8Ea^2R} \tag{Kdeb7}$$

As described in the preceding section, one of the first steps in the design of such a balance is to decide which forces or moments the balance should be designed to measure. In the above example, the desire was to measure the shear forces and bending moments communicated between the impeller and the drive shaft. Therefore, the balance needed to have some capability to deform in the shear and bending directions. One suitable design was the four post configuration shown in Figures 1 and 2. However, as a result of flexibility in these directions it would also have relatively low natural frequencies of oscillation in the lateral displacement mode and in the torsional mode. Therefore, it would clearly be advantageous for these two natural frequencies to be comparable. To achieve this first design criterion, a simple structural analysis shows the radius, R, where the posts are located should be approximately equal to the radius of gyration of the impeller mounted on the balance. Given that radius of gyration, R is therefore determined. The next step is then to decide on the desired natural torsional and lateral vibration frequencies which depend on a/λ^3 and therfore determine the magnitude of that quantity. Therefore, a low value of the aspect ratio, λ, is desirable; however, if λ were too small the strain gauges would be too close to the ends of the posts and this essentially sets a lower limit for λ of the order of 5. This in turn, given a desired natural frequency of torsion and lateral vibration, determines a. In this way the geometry of the posts and the balance becomes established. Of course, this geometry must also be evaluated using the expressions (Kdeb7) to determine the magnitudes of the strains that would result from the expected force magnitudes. At this stage the type of strain gauge required and the gauge factor of those strain gauges can be determined. If these are outside the realm of available strain gauges then the design needs to be reassessed and readjusted.