Conservation of Mass

Consider now the construction of the effective differential equations of motion for a disperse multiphase flow (such as might be used in a two-fluid model) assuming that an appropriate elemental volume can be identified. For convenience this elemental volume is chosen to be a *unit* cube with edges parallel to the x_1, x_2, x_3 directions. The mass flow of component N through one of the faces perpendicular to the i direction is given by $\rho_N j_{Ni}$ and therefore the net outflow of mass of component N from the cube is given by the divergence of $\rho_N j_{Ni}$ or

$$\frac{\partial(\rho_N j_{Ni})}{\partial x_i} \qquad \text{(Nbb1)}$$

The rate of increase of the mass of component N stored in the elemental volume is $\partial(\rho_N \alpha_N)/\partial t$ and hence conservation of mass of component N requires that

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{\partial(\rho_N j_{Ni})}{\partial x_i} = \mathcal{I}_N \qquad \text{(Nbb2)}$$

where $\mathcal{I}_N$ is the rate of transfer of mass to the phase N from the other phases per unit total volume. Such mass exchange would result from a phase change or chemical reaction. This is the first of several phase interaction terms that will be identified and, for ease of reference, the quantities $\mathcal{I}_N$ will termed the *mass interaction terms*.

Clearly there will be a continuity equation like (Nbb2) for each phase or component present in the flow. They will referred to as the Individual Phase Continuity Equations (IPCE). However, since mass as a whole must be conserved whatever phase changes or chemical reactions are happening it follows that

$$\sum_N \mathcal{I}_N = 0 \qquad \text{(Nbb3)}$$

and hence the sum of all the IPCEs results in a Combined Phase Continuity Equation (CPCE) that does not involve $\mathcal{I}_N$:

$$\frac{\partial}{\partial t}\left(\sum_N \rho_N \alpha_N\right) + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N j_{Ni}\right) = 0 \qquad \text{(Nbb4)}$$

or using equations (Nac4) and (Nac8):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\sum_N \rho_N \alpha_N u_{Ni}\right) = 0 \qquad \text{(Nbb5)}$$

Notice that only under the conditions of *zero* relative velocity in which $u_{Ni} = u_i$ does this reduce to the Mixture Continuity Equation (MCE) which is identical to that for an equivalent single phase flow of density ρ:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \qquad \text{(Nbb6)}$$

We also record that for one-dimensional duct flow the individual phase continuity equation (Nbb2) becomes

$$\frac{\partial}{\partial t}(\rho_N \alpha_N) + \frac{1}{A}\frac{\partial}{\partial x}(A \rho_N \alpha_N u_N) = \mathcal{I}_N \qquad \text{(Nbb7)}$$

where x is measured along the duct, $A(x)$ is the cross-sectional area, u_N, α_N are cross-sectionally averaged quantities and $A\mathcal{I}_N$ is the rate of transfer of mass to the phase N per unit length of the duct. The sum over the constituents yields the combined phase continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(A\sum_N \rho_N \alpha_N u_n\right) = 0 \tag{Nbb8}$$

When all the phases travel at the same speed, $u_N = u$, this reduces to

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}(\rho A u) = 0 \tag{Nbb9}$$

Finally we should make note of the form of the equations when the two components or species are intermingled rather than separated since we will analyze several situations with gases diffusing through one another. Then both components occupy the entire volume and the void fractions are effectively unity so that the continuity equation (Nbb2) becomes:

$$\frac{\partial \rho_N}{\partial t} + \frac{\partial(\rho_N u_{Ni})}{\partial x_i} = \mathcal{I}_N \tag{Nbb10}$$