Normal Shock Wave

Normal shock waves not only constitute a flow of considerable practical interest but also provide an illustrative example of the important role that relative motion may play in particle or droplet laden gas flows. In a frame of reference fixed in the shock, the fundamental equations for this steady flow in one Cartesian direction (x with velocity u in that direction) are obtained from equations (Nnb1) to (Nnb8) as follows. Neglecting any mass interaction ($\mathcal{I}_N = 0$) and assuming that there is one continuous and one disperse phase, the individual continuity equations (Nnb1) become

$$\rho_N \alpha_N u_N = \dot{m}_N = \text{constant} \tag{Nne1}$$

where $\dot{m}_C$ and $\dot{m}_D$ are the mass flow rates per unit area. Since the gravitational term and the deviatoric stresses are negligible, the combined phase momentum equation (Nnb7) may be integrated to obtain

$$\dot{m}_C u_C + \dot{m}_D u_D + p = \text{constant} \tag{Nne2}$$

Also, eliminating the external heat added ($\mathcal{Q} = 0$) and the external work done ($\mathcal{W} = 0$) the combined phase energy equation (Nnb8) may be integrated to obtain

$$\dot{m}_C(c_{vC}T_C + \frac{1}{2}u_C^2) + \dot{m}_D(c_{sD}T_D + \frac{1}{2}u_D^2) + pu_C = \text{constant} \tag{Nne3}$$

and can be recast in the form

$$\dot{m}_C(c_{pC}T_C + \frac{1}{2}u_C^2) + pu_C(1-\alpha_C) + \dot{m}_D(c_{sD}T_D + \frac{1}{2}u_D^2) = \text{constant} \tag{Nne4}$$

In lieu of the individual phase momentum and energy equations, we use the velocity and temperature relaxation relations (Nnd3) and (Nnd6):

$$\frac{Du_D}{Dt} = u_D \frac{du_D}{dx} = \frac{u_C - u_D}{t_u} \tag{Nne5}$$

$$\frac{DT_D}{Dt} = u_D \frac{dT_D}{dx} = \frac{T_C - T_D}{t_T} \tag{Nne6}$$

where, for simplicity, we confine the present analysis to the pure conduction case, $Nu = 2$.

Carrier (1958) was the first to use these equations to explore the structure of a normal shock wave for a gas containing solid particles, *a dusty gas* in which the volume fraction of particles is negligible. Under such circumstances, the initial shock wave in the gas is unaffected by the particles and can have a thickness that is small compared to the particle size. We denote the conditions upstream of this structure by the subscript 1 so that

$$u_{C1} = u_{D1} = u_1 \quad ; \quad T_{C1} = T_{D1} = T_1 \tag{Nne7}$$

The conditions immediately downstream of the initial shock wave in the gas are denoted by the subscript 2. The normal single phase gas dynamic relations allow ready evaluation of u_{C2}, T_{C2} and p_2 from u_{C1}, T_{C1} and p_1.

Unlike the gas, the particles pass through this initial shock without significant change in velocity or temperature so that

$$u_{D2} = u_{D1} \quad ; \quad T_{D2} = T_{D1} \tag{Nne8}$$

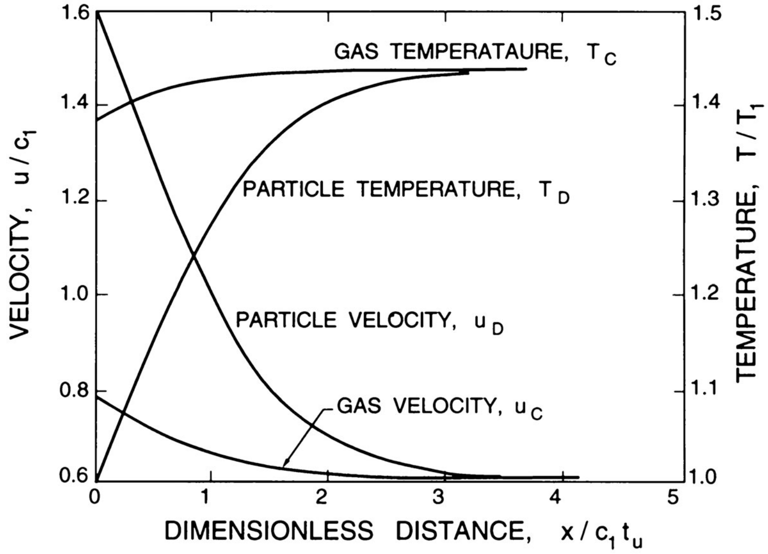

Figure 1: Typical structure of the relaxation zone in a shock wave in a dusty gas for $M_1 = 1.6$, $\gamma = 1.4$, $\xi = 0.25$ and $t_u/t_T = 1.0$. In the non-dimensionalization, c_1 is the upstream acoustic speed. Adapted from Marble (1970).

Consequently, at the location 2 there are now substantial velocity and temperature differences, $u_{C2} - u_{D2}$ and $T_{C2} - T_{D2}$, equal to the velocity and temperature differences across the initial shock wave in the gas. These differences take time to decay and do so according to equations (Nnd5) and (Nnd6). Thus the structure downstream of the gas dynamic shock consists of a relaxation zone in which the particle velocity decreases and the particle temperature increases, each asymptoting to a final downstream state that is denoted by the subscript 3. In this final state

$$u_{C3} = u_{D3} = u_3 \quad ; \quad T_{C3} = T_{D3} = T_3 \tag{Nne9}$$

As in any similar shock wave analysis the relations between the initial (1) and final (3) conditions, are independent of the structure and can be obtained directly from the basic conservation equations listed above. Making the small disperse phase volume approximation discussed in section (Nnc) and using the definitions (Nnc6), the relations that determine both the structure of the relaxation zone and the asymptotic downstream conditions are

$$\dot{m}_C = \rho_C u_C = \dot{m}_{C1} = \dot{m}_{C2} = \dot{m}_{C3} \quad ; \quad \dot{m}_D = \rho_D u_D = \xi \dot{m}_C \tag{Nne10}$$

$$\dot{m}_C(u_C + \xi u_D) + p = (1+\xi)\dot{m}_C u_{C1} + p_1 = (1+\xi)\dot{m}_C u_{C3} + p_3 \tag{Nne11}$$

$$(c_{pC}T_C + \frac{1}{2}u_C^2) + \xi(c_{sD}T_D + \frac{1}{2}u_D^2) = (1+\xi)(c_p T_1 + \frac{1}{2}u_1^2) = (1+\xi)(c_p T_3 + \frac{1}{2}u_3^2) \tag{Nne12}$$

and it is a straightforward matter to integrate equations (Nne5), (Nne6), (Nne10), (Nne11) and (Nne12) to obtain $u_C(x)$, $u_D(x)$, $T_C(x)$, $T_D(x)$ and $p(x)$ in the relaxation zone.

First, we comment on the typical structure of the shock and the relaxation zone as revealed by this numerical integration. A typical example from the review by Marble (1970) is included as figure 1. This shows the asymptotic behavior of the velocities and temperatures in the case $t_u/t_T = 1.0$. The nature of the relaxation processes is evident in this figure. Just downstream of the shock the particle temperature and velocity are the same as upstream of the shock; but the temperature and velocity of the gas has now changed and, over the subsequent distance, $x/c_1 t_u$, downstream of the shock, the particle temperature rises toward that of the gas and the particle velocity decreases toward that of the gas. The relative motion also causes a pressure rise in the gas, that, in turn, causes a temperature rise and a velocity decrease in the gas.

Clearly, there will be significant differences when the velocity and temperature relaxation times are not of the same order. When $t_u \ll t_T$ the velocity equilibration zone will be much thinner than the thermal relaxation zone and when $t_u \gg t_T$ the opposite will be true. Marble (1970) uses a perturbation analysis about the final downstream state to show that the two processes of velocity and temperature relaxation are not closely coupled, at least up to the second order in an expansion in ξ. Consequently, as a first approximation, one can regard the velocity and temperature relaxation zones as uncoupled. Marble also explores the effects of different particle sizes and the collisions that may ensue as a result of relative motion between the different sizes.

This normal shock wave analysis illustrates that the notions of velocity and temperature relaxation can be applied as modifications to the basic gas dynamic structure in order to synthesize, at least qualitatively, the structure of the multiphase flow.