Cavity Closure Models

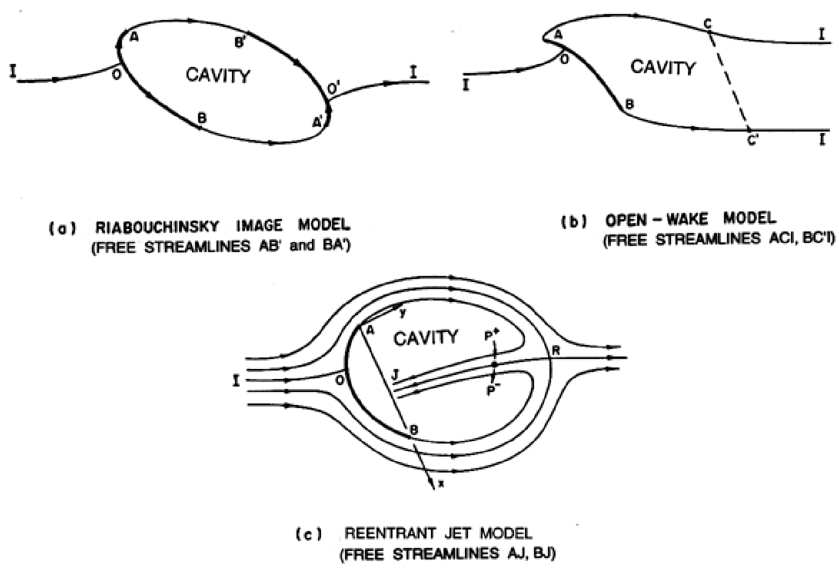

Figure 1: Closure models for the potential flow around an arbitrary body shape (AOB) with a fully developed cavity having free streamlines or surfaces as shown. In planar flow, these geometries in the physical or z-plane transform to the geometries shown in first figure of section (Nue).

Addressing first the closure problem, it is clear that most of the complex processes that occur in this region and that were described in section (Ntj) cannot be incorporated into a potential flow model. Moreover, it is also readily apparent that the condition of a prescribed free surface velocity would be violated at a rear stagnation point such as that depicted in the previous section. It is therefore necessary to resort to some artifact in the vicinity of this rear stagnation point in order to effect termination of the cavity. A number of closure models have been devised; some of the most common are depicted in Figures 1 and 2. Each has its own advantages and deficiences:

1. Riabouchinsky (1920) suggested one of the simpler models, in which an "image" of the body is placed in the closure region so that the streamlines close smoothly onto this image. In the case of planar or axisymmetric bodies appropriate shapes for the image are readily found; such is not the case for general three-dimensional bodies. The advantage of the Riabouchinsky model is the simplicity of the geometry and of the mathematical solution. Since the combination of the body, its image, and the cavity effectively constitutes a finite body, it must satisfy D'Alembert's paradox, and therefore the drag force on the image must be equal and opposite to that on the body. Also note that the rear stagnation point is no longer located on a free surface but has been removed to the surface of the image. The deficiences of the Riabouchinsky model are the artificiality of the image body and the fact that the streamlines downstream are an image of those upstream. The model would be more realistic if the streamlines downstream of the body-cavity system were displaced outward relative to their locations upstream of the body in order to simulate the effect of a wake. Nevertheless, it remains one of the most useful models, especially when the cavity is large, since the pressure distribution and therefore the force on the body is not substantially affected by the presence of the distant image body.

2. Joukowski (1890) proposed solving the closure problem by satisfying the dynamic free surface condition only up to a certain point on the free streamlines (the points C and C' in Figure 1) and then somehow

continuing these streamlines to downstream infinity, thus simulating a wake extending to infinity. This is known as the "open-wake model." For symmetric, pure-drag bodies these continuations are usually parallel with the uniform stream (Roshko 1954). Wu (1956, 1962) and Mimura (1958) extended this model to planar flows about lifting bodies for which the conditions on the continued streamlines are more complex. The advantage of the open-wake model is its simplicity. D'Alembert's paradox no longer applies since the effective body is now infinite. The disadvantage is that the wake is significantly larger than the real wake (Wu, Whitney, and Brennen 1971). In this sense the Riabouchinsky and open-wake models bracket the real flow.

3. The "reentrant jet" model, which was first formulated by Kreisel (1946) and Efros (1946), is also shown in Figure 1. In this model, a jet flows into the cavity from the closure region. Thus the rear stagnation point, R, has been shifted off the free surfaces into the body of the fluid. Moreover, D'Alembert's paradox is again avoided because the effective body is no longer simple and finite; one can visualize the momentum flux associated with the reentrant jet as balancing the drag on the body. One of the motivations for the model is that reentrant jets are often observed in real cavity flows, as discussed in section (Ntj). In practice the jet impacts one of the cavity surfaces and is reentrained in an unsteady and unmodeled fashion. In the mathematical model the jet disappears onto a second Riemann sheet. This represents a deficiency in the model since it implies an unrealistic removal of fluid from the flow and consequently a wake of "negative thickness." In one of the few detailed comparisons with experimental observations, Wu et al. (1971) found that the reentrant jet model did not yield results for the drag that were as close to the experimental observations as the results for the Riabouchinsky and open-wake models.

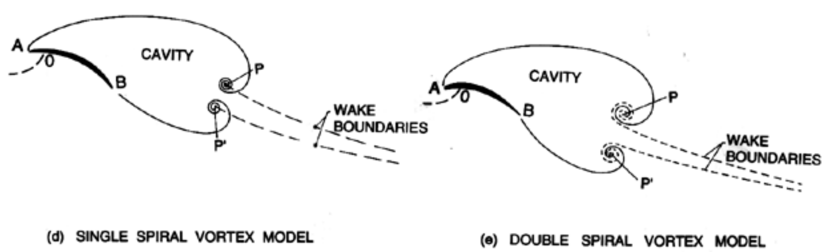

Figure 2: Two additional closure models for planar flow suggested by Tulin (1953, 1964). The free streamlines end in the center of the vortices at the points P and P' which are also the points of origin of the wake boundary streamlines on which the velocity is equal to U_∞.

4. Two additional models for planar, two-dimensional flow were suggested by Tulin (1953, 1964) and are depicted in Figure 2. In these models, termed the "single spiral vortex model" and the "double spiral vortex model," the free streamlines terminate in a vortex at the points P and P' from which emerge the bounding streamlines of the "wake" on which the velocity is assumed to be U_∞. The shapes of the two wake bounding streamlines are assumed to be identical, and their separation vanishes far downstream. The double spiral vortex model has proved particularly convenient mathematically (see, for example, Furuya 1975a) and has the attractive feature of incorporating a wake thickness that is finite but not as unrealistically large as that of the open-wake model. The single spiral vortex model has been extensively used by Tulin and others in the context of the linearized or small perturbation theory of cavity flows (see section (Nug)).

Not included in this list are a number of other closure models that have either proved mathematically difficult to implement or depart more radically from the observations of real cavities. For a discussion of these the reader is referred to Wu (1969, 1972) or Tulin (1964). Moreover, most of the models and much of the above discussion assume that the flow is steady. Additional considerations are necessary when modeling unsteady cavity flows (see section (Nul)).