Linearized Methods

When the body/cavity system is slender in the sense that the direction of the velocity vector is everywhere close to that of the oncoming uniform stream (except, perhaps, close to some singularities), then methods similar to those of thin airfoil theory (see, for example, Biot 1942) become feasible. The approximations involved lead to a more tractable mathematical problem and to approximate solutions in circumstances in which the only alternative would be the application of more direct numerical methods. Linear theories for cavity flows were pioneered by Tulin (1953). Though the methods have been extended to three-dimensional flows, it is convenient to begin by describing their application to the case of an inviscid and incompressible planar flow of a uniform stream of velocity, U_∞, past a single, streamlined cavitating body. It is assumed that the body is slender and that the wetted surface is described by $y = h(x)$ where $dh/dx \ll 1$. It is also assumed that the boundary conditions on the body and the cavity can, to a first approximation, be applied on the x-axis as shown in Figure 1. The velocity components at any point are denoted by $u = U_\infty + u'$ and v where the linearization requires that both u' and v are much smaller than U_∞. The appropriate boundary condition on the wetted surface is then

$$v = U_\infty \frac{dh}{dx} \tag{Nug1}$$

Moreover, the coefficient of pressure anywhere in the flow is given by $C_p \approx -2u'/U_\infty$, and therefore the boundary condition on a free streamline becomes

$$u' = \sigma U_\infty / 2 \tag{Nug2}$$

Finally, a boundary condition at infinity must also be prescribed. In some instances it seems appropriate to linearize about an x-axis that is parallel with the velocity at infinity. In other cases, it may be more appropriate and more convenient to linearize about an x-axis that is parallel with a mean longitudinal line through the body-cavity system. In the latter case the boundary condition at infinity is $w(z \to \infty) \to U_\infty e^{-i\alpha}$ where α is the angle of incidence of the uniform stream relative to the body-cavity axis.

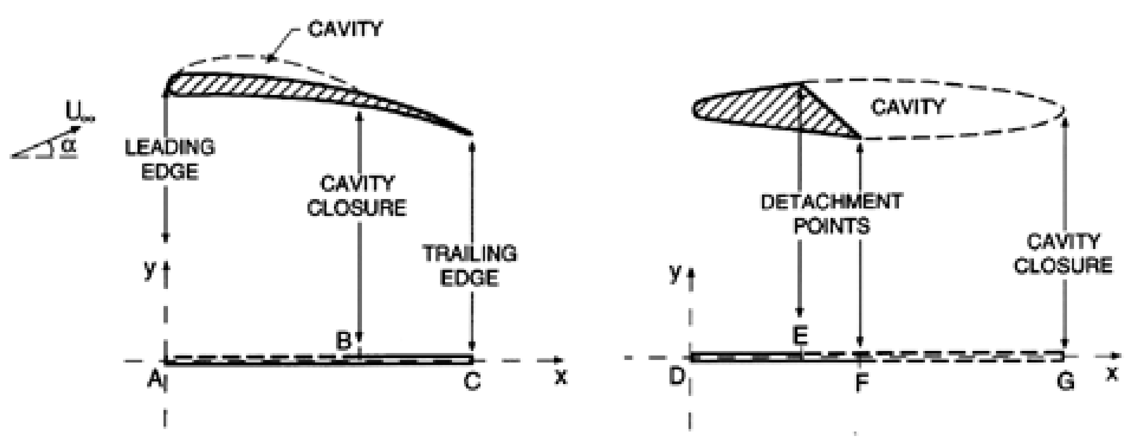

Figure 1: Examples of the linearized geometry (lower figures) for two planar cavity flows (upper sketches): a partially cavitating foil (left) and a supercavitating headform (right). Solid boundaries are indicated by the thick lines and the free streamlines by the thick dashed lines.

Even within the confines of this simple problem, several different configurations of wetted surface and free surface are possible, as illustrated by the two examples in Figure 1. Moreover, various types of singularity can occur at the end points of any segment of boundary in the linearized plane (points A through G in Figure 1). It is important that the solution contain the correct singular behavior at each of these points. Consider the form that the complex conjugate perturbation velocity, $w = u' - iv$, must take for each of the different types of singularity that can occur. Let $x = c$ be the location of the specific singularity under consideration. Clearly, then, a point like D, the stagnation point at a rounded nose or leading edge, must have a solution of the form $w \sim i(z-c)^{-1/2}$ (Newman 1956). On the other hand, a sharp leading edge from which a free surface detaches (such as A) must have the form $w \sim i(z-c)^{-1/4}$ (Tulin 1953). These results are readily derived by applying the appropriate conditions of constant v or constant u' on $\theta = 0$ and $\theta = 2\pi$.

The conditions at regular detachment points such as E or F (as opposed to the irregular combination of a detachment point and front stagnation point at A) should follow the conditions derived earlier for detachment points (section (Nuc)). If it is an abrupt detachment point, then w is continous and the singular behavior is $w \sim (z-c)^{1/2}$; on the other hand, if it is a smooth detachment point, both w and dw/dz must be continuous and $w \sim (z-c)^{3/2}$. At cavity closure points such as B or G various models have been employed (Tulin 1964). In the case of the supercavitating body, Tulin's (1953) original model assumes that the point G is a stagnation point so that the singular behavior is $w \sim (z-c)^{1/2}$; this is also the obvious choice under the conditions that u' is constant on $\theta = \pm\pi$. However, with this closure condition the circulation around the body-cavity system can no longer be arbitrarily prescribed. Other closure conditions that address this issue have been discussed by Fabula (1962), Woods and Buxton (1966), Nishiyama and Ota (1971), and Furuya (1975a), among others. In the case of the partial cavity almost all models assume a stagnation point at the point B so that the singular behavior is $w \sim (z-c)^{1/2}$. The problem of prescription of circulation that occurred with the supercavitation closure does not arise in this case since the conventional, noncavitating Kutta condition can be applied at the trailing edge, C.

The literature on linearized solutions for cavity flow problems is too large for thorough coverage in this text, but a few important milestones should be mentioned. Tulin's (1953) original work included the solution for a supercavitating flat plate hydrofoil with a sharp leading edge. Shortly thereafter, Newman (1956) showed how a rounded leading edge might be incorporated into the linear solution and Cohen, Sutherland, and Tu (1957) provided information on the wall effects in a tunnel of finite width. Acosta (1955) provided the first partial cavitation solution, specifically for a flat plate hydrofoil (see below). For a more recent treatment of supercavitating single foils, the reader is referred to Furuya and Acosta (1973).

It is appropriate to examine the linear solution to a typical cavity flow problem and, in the next section, the details for a cavitating flat plate hydrofoil will be given.

Many other types of cavitating flow have been treated by linear theory, including such problems as the effect of a nearby ocean surface. An important class of solutions is that involving cascades of foils, and these are addressed in section (Nui).