Effects of Vibration on Granular Flows

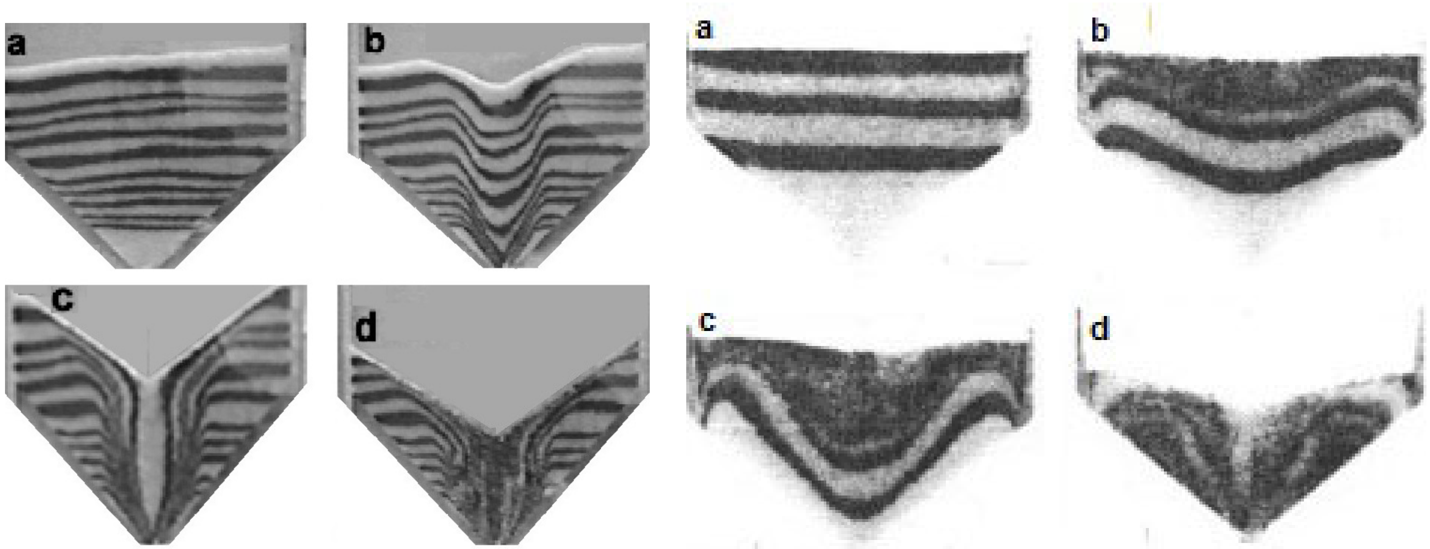

Figure 1: Four sequential images of the flow through a two-dimensional hopper: Left: Without imposed vibration. Right: With vertical vibration. Adapted from Hunt *et al.* (1999) and Wassgren *et al.* (2002).

The effects of vertical vibration on flow from wedge-shaped hoppers and flat bottom bins were first examined by Takahashi *et al.* (1968) and Suzuki *et al.* (1968). They reported the appearance of convection cells near the inclined wall boundaries of the hopper. In addition, the discharge rate was shown to increase with frequency at a fixed acceleration level, but that at the highest accelerations the discharge rate decreased significantly. Vibration also induced flow in bins that could not discharge under gravity alone. Lindemann and Dimon (2000) investigated flow from vertically oscillated funnels with small exit widths and wall angles. They found that the oscillation significantly affected how particles jam or mechanically arch at the exit. Both Lindemann and Dimon and Evesque and Meftah (1993) also report that the flow rate from vibrated hoppers decreases with increasing amplitude of oscillation.

Wassgren *et al.* (2002) studied the change in the flow patterns in a hopper caused by imposed vertical vibration; their typical observations are illustrated in Figure 1. Without oscillation the flow proceeds in the normal funnel flow manner as described in section (Npi). With vibration, the flow pattern depended on the magnitude of the vibrational acceleration. With the discharge blocked, two convection cells appear with particles moving up along the inclined walls of the hopper and down in the middle. With the blockage removed, the circulation pattern continues as the hopper discharges. The material continues to circulate as the hopper discharges until the hopper is completely emptied. The convection strength and direction depend strongly on the hopper wall angle: for hopper half-wall angles less than approximately 10 the convection cells are oriented downward at the walls while for angles greater than 10 particles move up at the walls.

The dimensionless flow rate from a vertically vibrating hopper as a function of the dimensionless oscillation velocity for a number of vibration accelerations (in g) and two different hoppers, A and B, is shown in

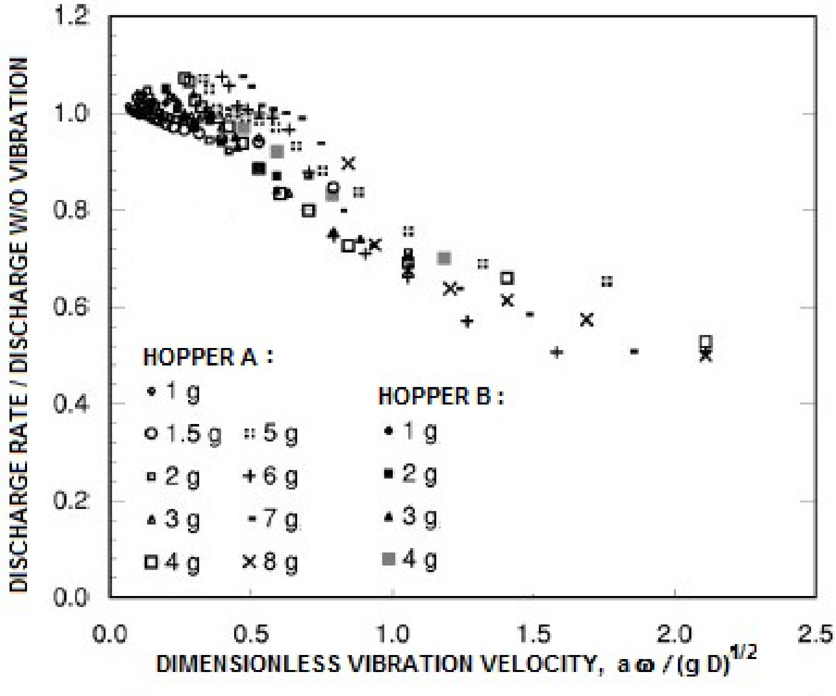

Figure 2: The dimensionless flow rate from a vertically vibrating hopper as a function of the dimensionless oscillation velocity for a number of vibration accelerations (in g) and two different hoppers, A and B. Adapted from Wassgren *et al.* (2002).

Figure 2. There appears to be little difference between the data for the two hoppers tested. For frequencies below $50Hz$ the discharge decreases with increasing acceleration level. At a fixed acceleration level, the decrease is most significant at the lowest frequencies. For the highest frequencies tested ($> 60Hz$) the vibration causes little change or leads to a small increase in the discharge.

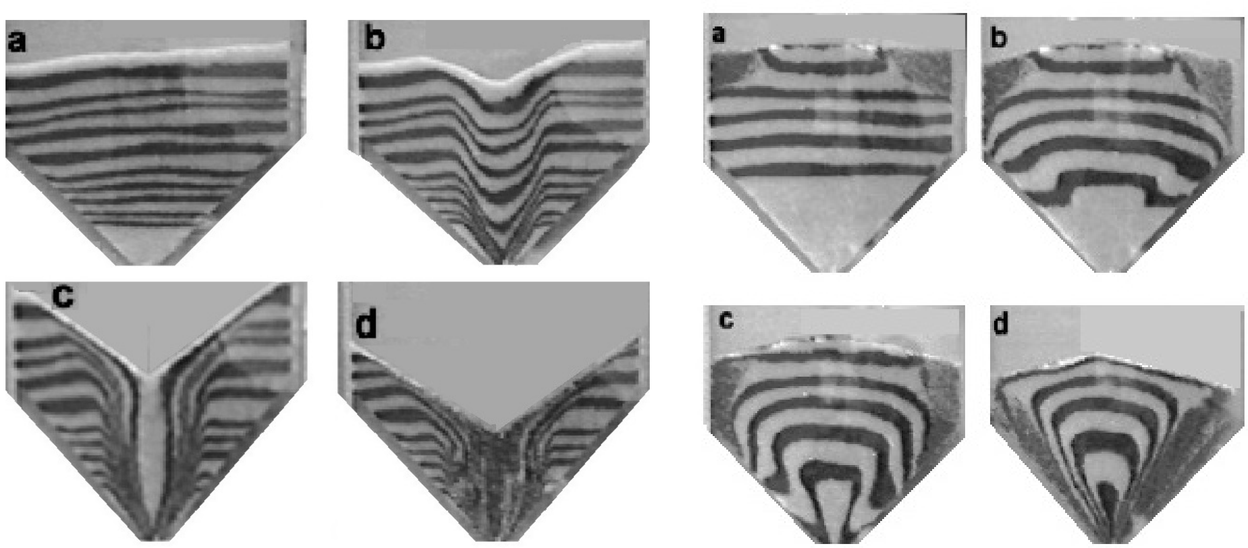

Figure 3: Four sequential images of the flow through a two-dimensional hopper subject to horizontal vibration: Left: Without imposed vibration. Right: With horizontal vibration. Adapted from Hunt *et al.* (1999).

Hunt *et al.* (1999) studied the effects of *horizontal* vibration as opposed to *vertical* vibration and found the effects of horizontal vibration to be quite different as demonstrated by the contrast between Figure 1 and Figure 3. Without oscillation the flow proceeds in the normal funnel flow manner as described above and in section (Npi). When the horizontal oscillation is initiated while the flow is blocked at the hopper discharge, convection cells form at the intersection of the hopper walls and the upper surface of the granular material. In these cells, the particles move up along the walls and down in the middle as seen in the right of Figure 3. The convection cells are a result of the granular bed dilation during free fall and the subsequent interaction with the hopper walls. Once the block at the exit is removed the flow along the sloping walls of the hopper proceeds faster than that in the middle and the flow pattern is entirely different than without the imposed oscillation. The measured discharge rates (scaled by the discharge rate without vibration) increase with the magnitude of the vibration as shown in Figure 4 where a and ω are the vibration amplitude and radian frequency and D is the opening width of the hopper.

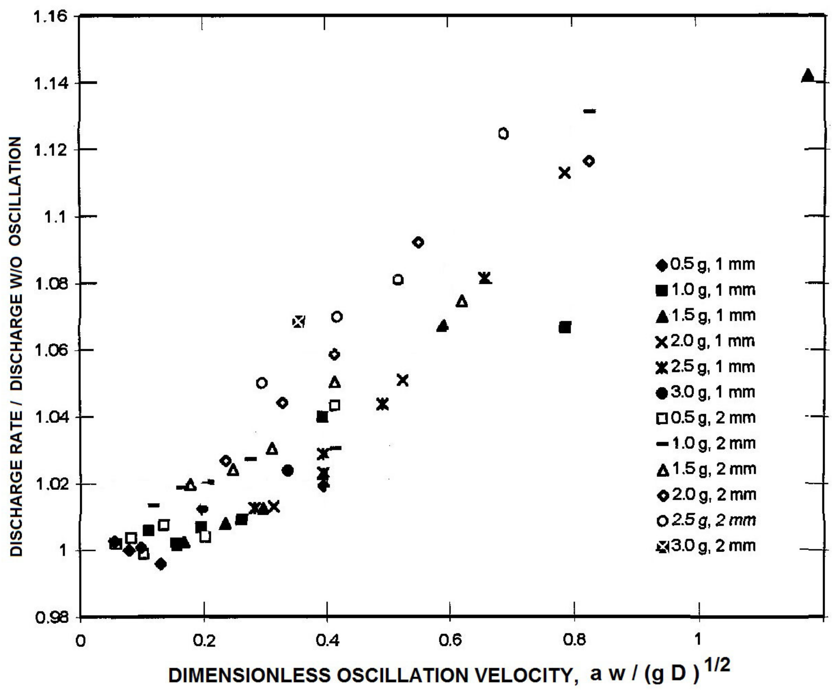

Figure 4: The dimensionless flow rate from a horizontally vibrating hopper as a function of the dimensionless oscillation velocity for two particle sizes ($1mm$ and $2mm$) and several oscillation accelerations as shown. Adapted from Hunt *et al.* (1999).

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Perhaps the first observations of the effects of vertical vibration on a bed of granular material were made by Faraday (1831) but subsequent studies were carried out by Douady *et al.* (1989), Fauve *et al.* (1989), Laroche *et al.* (1990), Thomas *et al.* (1989), Brennen *et al.* (1993), Pak *et al.* (1993 & 1995), Melo *et al.* (1994 & 1995), Wassgren (1997), Wassgren *et al.* (1996) and Wassgren *et al.* (2002) among others. We focus here on the response to vertical vibration (amplitude, a, frequency, f and radian frequency, $\omega = 2\pi f$) of a layer of granular material (particle diameter, d) of depth, h (at rest), in a box. The response depends on the dimensionless acceleration amplitude, $\Gamma = a\omega^2/g$.

As the acceleration amplitude is increased there is little response until, at some value just above $1g$ ($\Gamma = 1$), side wall convection cells appear at the walls of the box. Particles move down along the walls and up within the bulk of the bed. Simulations indicate that the convection cells are the result of the frictional contact between particles and the walls and the asymmetry of the particle/wall collision rate over an oscillation cycle(Wassgren 1997). As in the original experiments of Faraday (1831), these convection motions begin to form *heaps* as seen in Figure 5 [B]. Generally a single heap occurs either to one side of the box or in the center (Fauve *et al.* 1989). The heaping is caused by a slow particle convection pattern in which particles avalanche down the free surface of the heap, are subducted into the mass at its lowest point, and then recirculate internally back to the peak usually along a vertical wall. Wassgren (1997) and Wassgren *et al.* (2002) describe how a deep bed of $d = 1.3mm$ glass beads exhibits heaping above a critical Γ of about 1.2 (see also Evesque and Rajchenbach 1989).

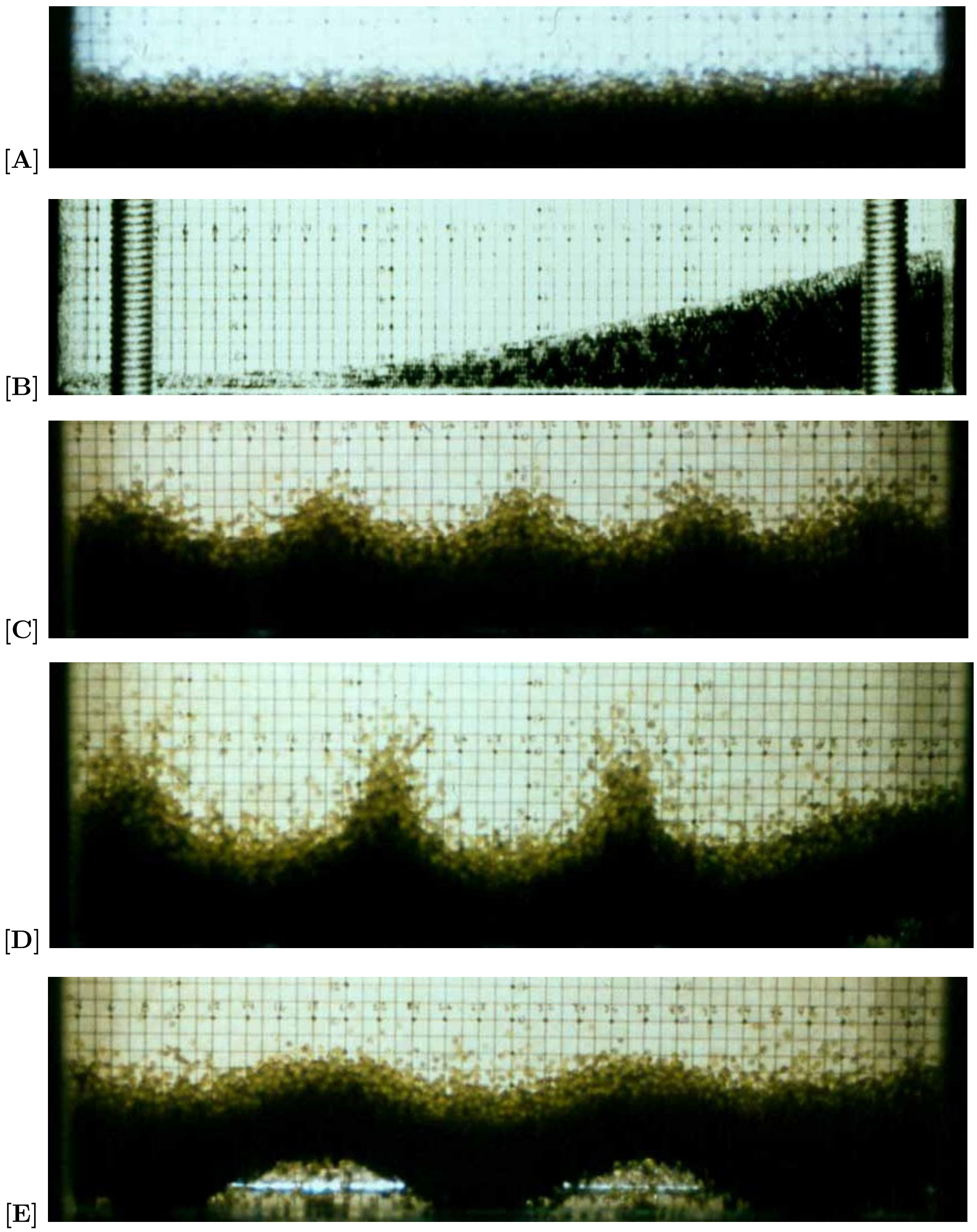

Figure 5: Flow patterns in a layer of $1.3mm$ diameter glass beads, 18 diameters deep, subject to vertical vibration: [A] At rest [B] Heaping [C] $f/2$ waves at $f = 20Hz$ $\Gamma = 3.3$ [D] $f/4$ waves at $f = 20Hz$ $\Gamma = 6.2$ [E] kinks at $f = 30Hz$ $\Gamma = 8.5$. Adapted from Wassgren (1997).

At larger accelerations, experiments reveal that the behavior of a particle bed is different for shallow and deep beds. When a shallow bed less than 6 particle diameters deep is subjected to oscillations with amplitudes greater than approximately $2g$, the particles in the container become fluidized and do not display coordinated movement. In this shallow bed regime three distinct sub-states are observed that differ in the degree of coherence in the particle motions. Their occurrence depends on the bed depth and the acceleration level and the transitions between the states are gradual and not well-defined.

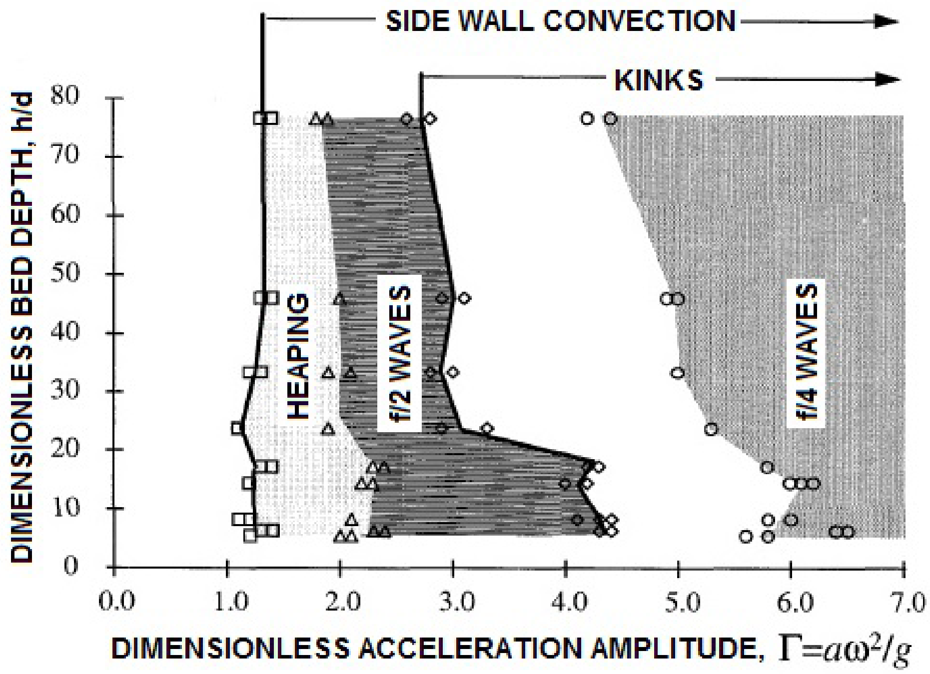

Figure 6: Flow regime chart for the vibration patterns due to vertical vibration of a deep bed of 1.3mm diameter glass beads. Adapted from Wassgren (1997).

In contrast, when the layer is more than 6 particle diameters deep, the particles move coherently and the deep bed behaves as a single inelastic mass. The transition from the deep bed state to the shallow bed state is characterized by a sudden expansion of the bed that occurs at a critical acceleration amplitude that depends on the bed depth and particle characteristics (Brennen *et al.* 1993). The simulations of Wassgren (1997) indicate that when the particle fluctuating kinetic energy is dissipated completely each oscillation cycle, the bed remains in the deep bed state. If the energy is not completely dissipated, a shallow bed state results.

At a higher critical Γ standing waves (known as *Faraday waves*) appear on the free surface of the bed (Figure 5 [C]). For $2.2 < \Gamma < 3.5$ the wave frequency is one-half the forcing frequency and the waves are termed $f/2$ waves. However, for $\Gamma > 5.5$ and up to at least 7.8 the wave frequency is one-quarter of the forcing frequency (referred to as $f/4$ waves) with, presumably, further bifurcations at higher frequencies (see below). For $\Gamma > 3.5$ neighboring regions of the particle bed can oscillate out-of-phase, forming *kink waves* (Figure 5 [E]) with counter-rotating *kink convection cells* bracketing each wave node. As seen in Figure 6 these Γ ranges do vary somewhat with layer thickness and particle characteristics. However, the ranges are similar to the measurements of Melo *et al.* (1995) who used beds of bronze spheres.

Brennen *et al.* (1993) placed a lightweight floating lid on top of a granular bed in order to damp out the waves and observe the dynamic bifurcations move closely. As the vibration amplitude is increased the bed first expands gradually as exemplified by the case of Figure 7. Then , at some critical Γ, there is a sudden expansion or bifurcation to a different but stable state. Thus the bed behaves in a manner similar to a single ball bouncing on a vertically vibrating plate (Holmes 1982). More birfurcations occur at higher critical acceleration levels.

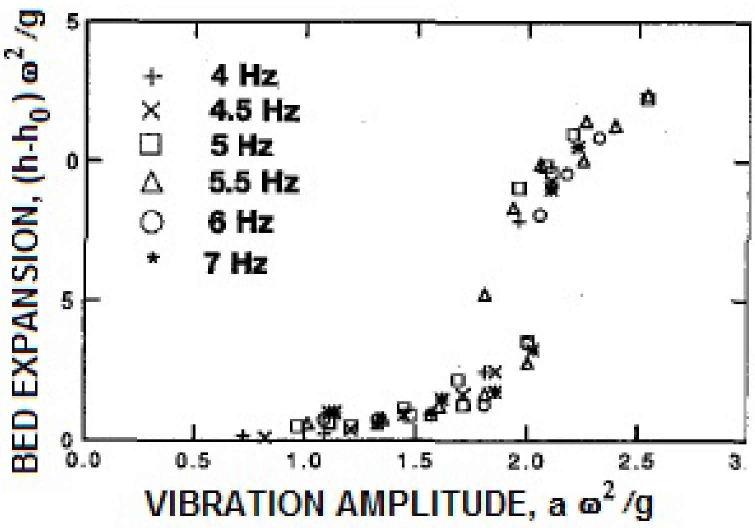

Figure 7: Example of the sudden expansion of a granular bed subject to vertically vibration. Adapted from Brennen *et al.* (1993).