Rapid Granular Flow

Despite the uncommon occurence of truly rapid granular flow, it is valuable to briefly review the substantial literature of analytical results that have been generated in this field. At high shear rates, the inertia of the random motions that result from particle-particle and particle-wall collisions becomes a key feature of the rheology. Those motions can cause a dilation of the material and the granular material begins to behave like a molecular gas. In such a flow, as in kinetic theory, the particle velocities can be decomposed into time averaged and fluctuating velocity components. The energy associated with the random or fluctuating motions is represented by the granular temperature, T, analogous to the thermodynamic temperature. Various granular temperatures may be defined depending on whether one includes the random energy associated with rotational and vibrational modes as well as the basic translational motions. The basic translational granular temperature used herein is defined as

$$T = \frac{1}{3}\left(< \grave{U}_1^2 > + < \grave{U}_2^2 > + < \grave{U}_3^2 > \right) \qquad \text{(Npj1)}$$

where $\grave{U}_i$ denotes the fluctuating velocity with a zero time average and $< >$ denotes the ensemble average. The kinetic theory of granular material is complicated in several ways. First, instead of tiny point molecules it must contend with a large solids fraction that inhibits the mean free path or flight of the particles. The large particle size also means that momentum is transported both through the flight of the particles (the *streaming* component of the stress tensor) and by the transfer of momentum from the center of one particle to the center of the particle it collides with (the *collisional* component of the stress tensor). Second, the collisions are inelastic and therefore the velocity distributions are not necessarily Maxwellian. Third, the finite particle size means that there may be a significant component of rotational energy, a factor not considered in the above definition. Moreover, the importance of rotation necessarily implies that the communication of rotation from one particle to another may be important and so the tangential friction in particle-particle and particle-wall collisions will need to be considered. All of this means that the development of a practical kinetic theory of granular materials has been long in development.

Early efforts to construct the equations governing rapid granular flow followed the constructs of Bagnold (1954); though his classic experimental observations have recently come under scrutiny (Hunt *et al.* 2002), his qualitative and fundamental understanding of the issues remains valid. Later researchers, building on Bagnold's ideas, used the concept of granular temperature in combination with heuristic but insightful assumptions regarding the random motions of the particles (see, for example, McTigue 1978, Ogawa *et al.* 1980, Haff 1983, Jenkins and Richman 1985, Nakagawa 1988, Babic and Shen 1989) in attempts to construct the rheology of rapid granular flows. Ogawa *et al.* (1978, 1980), Haff (1983) and others suggested that the global shear and normal stresses, τ_s and τ_n, are given by

$$\tau_s = f_s(\alpha)\rho_S\dot{\gamma}T^{\frac{1}{2}} \qquad \text{and} \qquad \tau_n = f_n(\alpha)\rho_S T \qquad \text{(Npj2)}$$

where f_s and f_n are functions of the solid fraction, α, and some properties of the particles. Clearly the functions, f_s and f_n, would have to tend to zero as $\alpha \rightarrow 0$ and become very large as α approaches the maximum shearable solids fraction. The constitutive behavior is then completed by some relation connecting T, α and, perhaps, other flow properties. Though it was later realized that the solution of a *granular energy* equation would be required to determine T, early dimensional analysis led to speculation that the granular temperature was just a local function of the shear rate, $\dot{\gamma}$ and that $T^{\frac{1}{2}} \propto D\dot{\gamma}$. With some adjustment in f_s and f_n this leads to

$$\tau_s = f_s(\alpha)\rho_S D^2\dot{\gamma}^2 \qquad \text{and} \qquad \tau_n = f_n(\alpha)\rho_S D^2\dot{\gamma}^2 \qquad \text{(Npj3)}$$

which implies that the effective friction coefficient, τ_s/τ_n should only be a function of α and the particle characteristics.