Molecular Effects

When the mean free path of the molecules in the surrounding fluid, λ, becomes comparable with the size of the particles, the flow will clearly deviate from the continuum models, that are only relevant when $\lambda \ll R$. The Knudsen number, $Kn = \lambda/2R$, is used to characterize these circumstances, and Cunningham (1910) showed that the first-order correction for small but finite Knudsen number leads to an additional factor, $(1 + 2AKn)$, in the Stokes drag for a spherical particle. The numerical factor, A, is roughly a constant of order unity (see, for example, Green and Lane 1964).

When the impulse generated by the collision of a single fluid molecule with the particle is large enough to cause significant change in the particle velocity, the resulting random motions of the particle are called *Brownian motion* (Einstein 1956). This leads to diffusion of solid particles suspended in a fluid. Einstein showed that the diffusivity, D, of this process is given by

$$D = kT/6\pi\mu_C R \tag{Ned1}$$

where k is Boltzmann's constant. It follows that the typical *rms* displacement of the particle in a time, t, is given by $(kTt/3\pi\mu_C R)^{\frac{1}{2}}$. Brownian motion is usually only significant for micron- and sub-micron-sized particles. The example quoted by Einstein is that of a 1 μm diameter particle in water at $17°C$ for which the typical displacement during one second is 0.8 μm.

A third, related phenomenon is the response of a particle to the collisions of molecules when there is a significant temperature gradient in the fluid. Then the impulses imparted to the particle by molecular collisions on the hot side of the particle will be larger than the impulses on the cold side. The particle will therefore experience a net force driving it in the direction of the colder fluid. This phenomenon is known as *thermophoresis* (see, for example, Davies 1966). A similar phenomenon known as *photophoresis* occurs when a particle is subjected to nonuniform radiation. One could include in this list the Bjerknes forces described in the section (Nfe) since they constitute *sonophoresis*, namely forces acting on a particle in a sound field.