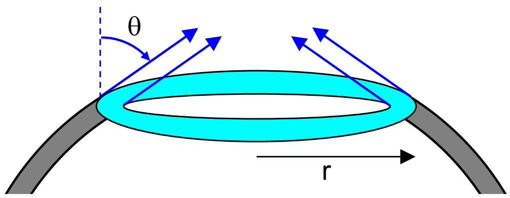
Solution to Problem 108C

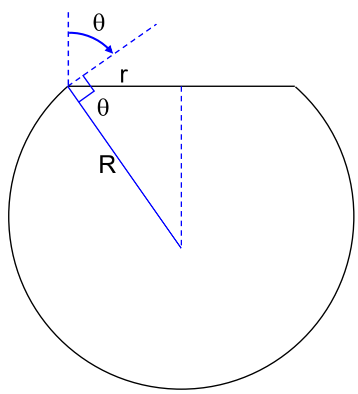
A soap bubble hangs from a horizontal circular ring of radius r equal to 3cm. The mass of the soapy water comprising the bubble is $m = .0014\ kg$. It is assumed that the bubble is spherical and any contact angle effects at the junction of the ring are negligible.

[1] The weight of the soap film must balance the component of the surface tension force, S. Note that there are two surfaces to consider.



$$\begin{aligned} mg &= 2\left[2\pi r S\cos\theta\right] \\ \theta &= \cos^{-1}\left[\frac{mg}{4\pi r S}\right] \\ &= \cos^{-1}\left[\frac{(0.0014\ kg)(9.81\ m/s^2)}{4\pi(0.03\ m)(0.05\ kg/s^2}\right] \\ \rightarrow \theta &\approx 43.3^\circ \end{aligned}$$

[2] The radius of the soap bubble follows from the geometry:



$$\begin{aligned} \cos\theta &= \frac{r}{R} \\ R &= \frac{r}{\cos\theta} \\ &= \frac{3\ cm}{\cos 43.3^\circ} \\ \rightarrow R &\approx 4.12\ cm \end{aligned}$$

[3] The thickness of the soap bubble follows from the density of the soapy water ($\rho = 1000\ kg/m^3$) and the mass ($m = .0014\ kg$). By assuming $t \ll R$, the approximate volume is given by the (surface area)×(thickness). The surface area follows from integration:

$$A(\theta) = 2\pi R^2 \int_{-\pi/2}^{\theta} \cos\phi\ d\phi$$

$$
\begin{aligned}
&= 2\pi R^2 \left(1+\sin\theta\right) \\
\therefore m &= \rho V \approx \rho A(\theta) t \\
\rightarrow t &= \frac{m}{\rho A(\theta)} = \frac{0.0014\ kg}{(1000\ kg/m^3)(2\pi)(0.0412\ m)^2(1+\sin(43.3^\circ))} \\
t &= 7.78\times 10^{-5}\ m
\end{aligned}
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