Problem 116A

In cylindrical coordinates, (r, θ, z), the equations of motion for an inviscid fluid, Euler's equations, become:

$$\rho \left[\frac{Du_r}{Dt} - \frac{u_\theta^2}{r} \right] = -\frac{\partial p}{\partial r} + f_r$$

$$\rho \left[\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} \right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta$$

$$\rho \frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + f_z$$

where u_r, u_θ, u_z are the velocities in the r, θ, z directions, p is the pressure, ρ is the fluid density and f_r, f_θ, f_z are the body force components. The Lagrangian derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$$

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A cylinder is rotated at a constant angular velocity denoted by Ω. The cylinder contains a compressible fluid which rotates with the cylinder so that the fluid velocity at any point is $u_\theta = \Omega r$ ($u_r = u_z = 0$). If the density of the fluid, ρ , is related to the pressure, p, by the polytropic relation

$$p = A\rho^k$$

where A and k are known constants, find the pressure distribution, $p(r)$, assuming that the pressure, p_0, at the center ($r = 0$) is known. Neglect all body forces.