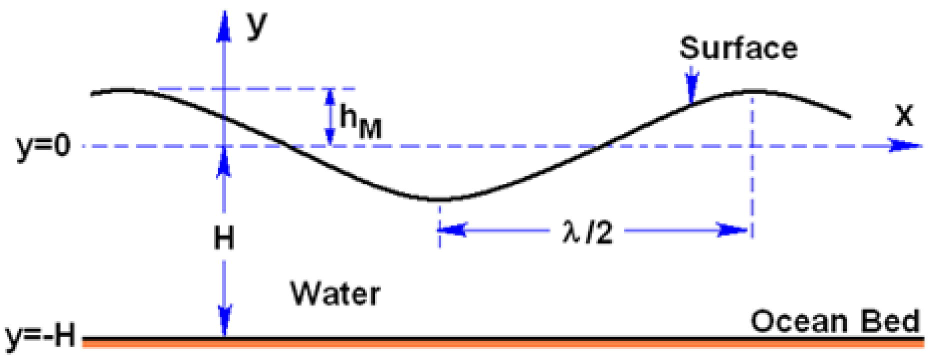
## Problem 130E

This problem is concerned with travelling waves on an "ocean" of **finite** depth, $H$:




The appropriate velocity potential for this flow (assuming it is irrotational) is of the form:

$$\phi = \left(Ae^{ny} + Be^{-ny}\right)\sin\left(nx - \Omega t\right)$$

where $n$ is the wavenumber (wavelength, $\lambda = 2\pi/n$), $\Omega$ is the radian frequency ($radians/s$) and $A$ and $B$ are constants initially undetermined. The axis, $y = 0$, is located at the mean position of the ocean surface and the wave amplitude, $h_M$, is sufficiently small so that the values of $v$ and $\partial\phi/\partial t$ at the surface can be approximated by the values of $v$ and $\partial\phi/\partial t$ at $y = 0$.

By applying two boundary conditions at the ocean surface (one kinematic, one dynamic) and one boundary condition on the ocean bottom find

1. Expressions for $A$ and $B$ in terms of $n$, $\Omega$, $H$ and $h_M$.
2. The equation for the shape, $y = h(x,t)$, of the ocean surface.
3. The wave velocity, $c$, in terms of $\lambda$, $H$ and $g$ (acceleration due to gravity). Does the wave velocity increase or decrease as the depth, $H$, decreases?

[PS. Close inspection of the assumptions made in this problem would show that the solution is only valid when $H \ll \lambda$.]