Problem 267A

This problem is concerned with assessment of the spatial stability characteristics of a simple parallel boundary layer flow given by

$$\frac{u}{U} = \frac{y}{\delta} + \left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3 \quad \text{for} \quad 0 < y < \delta$$

$$\frac{u}{U} = 1 \quad \text{for} \quad y > \delta$$

where, clearly, U is the constant and uniform velocity exterior to the boundary layer and δ is the boundary layer thickness.

Neglecting viscous effects on the stability and using Rayleigh's equation rather than the more general Orr-Sommerfeld equation, describe in detail how you would compute the amplification rate of spatially periodic travelling wave disturbances (the amplitude is not growing in time at a particular spatial location). Find an appropriate non-dimensional form for Rayleigh's equation in this case. What is an appropriate non-dimensional frequency for a disturbance? What is an appropriate non-dimensional wavelength? What is an appropriate non-dimensional amplification rate? What non-dimensional parameters will the non-dimensional amplification rate be a function of?

Prepare a detailed block diagram for the computer program needed to compute the results of this stability analysis.

What additional non-dimensional parameter is introduced if the viscous terms are reintroduced so that you are now solving the Orr-Sommerfeld equation?