

Problem 270E

In our synthesis of turbulent pipe flow, we used the universal velocity profile which assumes a uniform shear stress (equal to the wall shear stress) throughout the flow. In fact, the shear stress in cylindrical pipe flow varies linearly with radius. Investigate this discrepancy, by constructing the differential equation for $\bar{u}(y)$ (y is the distance from the wall, $y = R - r$, where r is the radial position in the pipe and R is the radius of the pipe). Assume a mixing length which varies linearly with radial position, is zero at the wall and is determined by a universal constant, κ . Assume the shear stress varies linearly with the radius. Include the laminar viscous contribution to the shear stress.

Find the differential equation which must be solved to determine $\bar{u}(y)$.