

Problem 271A

The velocity profile in a turbulent boundary layer of incompressible fluid on a flat plate ($U = \text{constant}$) is to be approximated by the form:

$$u/U = (y/\delta)^{\frac{1}{7}}$$

[Disregard the fact that this does not exactly satisfy one of the constraints usually imposed on laminar boundary layer profiles namely that du/dy should tend to zero as y tends to δ]. Find the profile parameter α for this profile. If the wall shear stress, τ_w , for this turbulent profile is assumed to be given by the empirical formula

$$\tau_w = 0.023\rho U^2(\nu/\delta U)^{\frac{1}{4}}$$

where ρ and ν are the fluid density and kinematic viscosity, then solve the resulting Karman momentum integral equation to obtain an expression for the thickness of the boundary layer, δ , as a function of distance, x , along the plate. Assume that the layer first becomes turbulent at $x = x_0$ where the thickness is δ_0 .

[Do not use $\tau_w = \mu(du/dy)_{y=0}$ which is inappropriate in turbulent boundary layer calculations.]