

Problem 274B

In 1844, to commemorate a visit by the Tsar of Russia, the Duke of Devonshire, the greatest landowner in Britain, wished to construct the tallest fountain ever built in the grounds of his great house at Chatsworth in Derbyshire. He employed the renowned engineer Joseph Paxton to build what was to become known as the Emperor Fountain. That fountain remains the tallest, gravity-fed fountain in the world, with a maximum height of $90.2m$ above the pond into which it falls. What Paxton did was to excavate a massive eight acre lake on a nearby hill such that the lake surface was $120m$ above the afore-mentioned pond. The pipe to the fountain was $800m$ long and had an internal diameter of $0.381m$. (Paxton knew that to maximize the height of the fountain he would have to make the pipe diameter large.) The result was that the maximum flow rate through the pipe (when the control valve was fully opened) was $15000 \text{ liters}/\text{min}$. Questions:

1. Using the above information find the friction factor for Paxton's pipe.
2. Find the Reynolds number for the flow in Paxton's pipe assuming a water temperature of $15^\circ C$ so that the kinematic viscosity of the water $1.16 \times 10^{-6} \text{ m}^2/\text{s}$.
3. What kind of flow is occurring in Paxton's pipe?
4. Using the answers to the first two questions estimate the typical height of the roughnesses in the interior surface of Paxton's pipe.
5. Assuming the same friction factor, what would the maximum height of the fountain have been if Paxton had used a pipe with a half of the above diameter?