

Problem 310A

Consider a straight pipe filled with incompressible liquid. The walls of the pipe are elastic so that the cross-sectional area, A , changes with the internal pressure, p , according to the relation:

$$A = A_0 + A_1 p$$

Thus the pipe may have different cross-sectional areas at different axial positions depending on the internal pressure at each position. Find the speed of propagation, c , of a small pressure wave travelling along the pipe assuming A_0 and A_1 are known constants and that $A_1 p$ is always small compared with A_0 so that simple binomial expansions may be used (see note). Give your answer in terms of A_0 , A_1 and the density, ρ , of the liquid. Note that if $b \ll a$ then:

$$(a + b)^n \approx a^n + n a^{n-1} b$$